

are precisely equivalent, gravity must bend light, by a precise amount that could be calculated. This was not entirely a startling suggestion: Newton's theory, based on the idea of light as a stream of tiny particles, also suggested that a light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exactly twice as great as it is according to Newton's theory.

When the bending of starlight caused by the gravity of the Sun was measured during a solar eclipse in 1919, (fig-20(b)) and found to match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

QUESTIONS :

Note :- The answers to the questions at the end of the Chapter are given below. For questions see last page.

Q. 5.1 :- Tangential velocity

- (1) It is the linear velocity of a particle moving along a curve or a circle directed along the tangent at any pt. on the curve.
- (2) It is denoted by v_T .
- (3) Its formula is : $v = r\omega$.
Also: $v = \frac{s}{t}$.
- (4) Its unit is m/s.

Angular velocity

- (1) It is the rate of change of angular displacement of a particle moving along a circular path.
- (2) It is denoted by ' ω '.
- (3) Its formulae are: $\omega = \frac{\theta}{\Delta t}$
Also: $\omega = \frac{v}{r}$.
- (4) Its unit is rad/s.

Q.5.2 :- "The force which is needed to move a body along a circular path is called the centripetal force".

To move an object in a circular path, its direction needs to be changed at every point. To change the direction of motion continuously, a continuous perpendicular force is required. This force is known as centripetal force. It is directed along the radius towards the centre of the circle.

Q.5.3 :- "A property of a body to resist its uniform circular motion is called the moment of inertia".

Moment of inertia is a characteristic of the configuration of the body w.r.t. the chosen axis. It depends on the mass of the body and its distribution w.r.t. the axis of rotation of the body.

For a rigid body consisting of particles of mass m_1, m_2, \dots, m_n situated at distances r_1, r_2, \dots, r_n from the axis of rotation, the moment of inertia is given by:

$$I = \sum_{i=1}^n m_i r_i^2$$

The moment of inertia plays the same role in angular motion as that of mass in linear motion. It is also called measure of rotational inertia in angular motion.

Q.5.4 :- Angular Momentum :- "It is the vector product of position vector ' \vec{r} ' and linear momentum ' \vec{p} ' i. e.

$$\vec{L} = \vec{r} \times \vec{p}$$

It is a quantity similar to linear momentum for angular motion. As linear momentum is a product of mass and linear velocity, similarly angular momentum can also be defined as the product of their counterparts i.e., moment of inertia 'I' and angular velocity ' $\vec{\omega}$ '.

Law of Cons. of angular Momentum :-

"The total angular momentum of all the bodies in a system remains constant if no external torque acts on the system." This is known as law of conservation of angular momentum. It can be expressed as : $L = I_1 \omega_1 = I_2 \omega_2 = \text{const}$, ...

If 'I' of a body decreases, its ' ω ' increases, so that their product remains constant.

Q. 5.5 :- We know that : $\vec{L} = \vec{r} \times \vec{p}$, and its magnitude is given by : $L = r p \sin \theta = m v r \sin \theta$ where ' θ ' is the angle between \vec{r} and \vec{v} . In case of circular orbital motion, $\theta = 90^\circ$, between \vec{r} and \vec{v} . Hence : $L_0 = m v r \sin 90^\circ = m v r$
($\because \sin 90^\circ = 1$)
 $\therefore L_0 = m v r$

Q. 5.6 :- We know that min. velocity to put a satellite into orbit is : $v_0 = \sqrt{gR}$. AS low flying Earth satellite has acc, $g = 9.8 \text{ m/s}^2$ Hence, putting the values of ' g ' and $R = 6400000 \text{ m}$,

In above eq. we get: $v_0 = \sqrt{9.8 \times 6400000} = 7.9 \frac{\text{km}}{\text{s}}$
 This is the min. velocity necessary to put a satellite into orbit around the earth.

Q.5.7 :- (i) Angular momentum :- As angular momentum ' \vec{L} ' is defined as: $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$.

Being cross product of \vec{r} and \vec{v} , its direction is perpendicular to the plane formed by \vec{r} and \vec{v} and given by right hand rule. Its direction is along the axis of rotation.

(ii) Angular Velocity :- The direction of angular velocity is taken along the axis of rotation given by right hand rule. According to this rule for counter clock-wise rotation, the direction of angular velocity, is outward along the axis of rotation.

Q.5.8 :- Pl. Study: "Weightlessness in satellites and gravity free system."

Q.5.9 :- The mud clinging to the tyre of a moving bicycle flies in a direction tangent to the tyre.

When the bicycle is moving, each pt. on the tyre including the mud clinging to the tyre has a tangential velocity v , as well as angular velocity ' ω '. A centripetal force " $\frac{mv^2}{r}$ " must be provided to the mud to keep it on the tyre. The adhesive force of mud to the surface of the tyre provides this inward force. When velocity of the cycle increases, the required

force necessary to keep the mud on the tyre increases. When required centripetal force exceeds the adhesive force, the mud is separated from the tyre and flies off the tyre in a direction tangent to the surface of the tyre.

Q. 5.10 :- The velocity of the disc on reaching the bottom of the inclined plane is given by the following eq.

$$v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}} \times \sqrt{gh} \longrightarrow (1)$$

and for a hoop, the velocity at the bottom of inclined plane is :

$$v' = \sqrt{gh} \longrightarrow (2)$$

From eq. (1) & (2) : $v = \sqrt{\frac{4}{3}} \times v' \Rightarrow \underline{v = 1.15v'}$

Hence, the speed of the disc is $v > v'$.

Q. 5.11 :- Before lifting off the diving board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about an axis. The moment of inertia is considerably reduced to a new value I_2 , when the legs and arms are drawn into closed tuck position. As angular momentum is conserved, so :

$$I_1 \omega_1 = I_2 \omega_2$$

where ' ω_1 ' & ' ω_2 ' are the angular speeds before and after diving. Hence the diver spins faster when its moment of inertia becomes smaller.

Q. 5.12 :- Using law of conservation of angular momentum :

$$I_i \omega_i = I_f \omega_f$$

With out-stretched arms, the moment of inertia is

' I ', and his angular speed ω_i . The product " $I\omega_i$ " is the angular momentum which remains constant. When he pulls the dumbbells towards his chest, his moment of inertia ' I_2 ' decreases. Now he spins faster as ' ω_f ' increases to keep ' $I_2\omega_f = \text{const.}$ '

Q. 5.13 :- A geo-stationary satellite covers 120° of longitude, so the whole of the Earth's surface for global transmission can be covered by three correctly positioned geo-stationary satellites.

NUMERICAL PROBLEMS

Note :- The solutions to the numerical problems are given below. For Problems see last page.

P. 5.1 :- As given :- Diameter of beam = arc length = $s = 2.5 \text{ m}$
Distance of moon = radius of circle = $r = 3.8 \times 10^8 \text{ m}$

$$\theta = ? \quad \text{Using: } s = r\theta, \quad \theta = \frac{s}{r} = \frac{2.5}{3.8 \times 10^8}$$

$$\therefore \theta = 6.6 \times 10^{-9} \text{ rad.} \quad = 6.6 \times 10^{-9} \text{ rad}$$

P. 5.2 :- As given :- Initial angular velocity = $\omega_i = 0$

Final angular velocity = $\omega_f = 45 \text{ rev/min}$

$$\text{or } \omega_f = \frac{45 \times 2\pi}{60} = \frac{45 \times 2 \times 3.14}{60}$$

$$\therefore \omega_f = 4.71 \text{ rad s}^{-1}$$

$$t = 1.6 \text{ s}, \quad \alpha = \text{ang. acc.} = ?$$

using the eq. $\omega_f = \omega_i + \alpha t$

Putting values: $4.71 = 0 + \alpha(1.6) \Rightarrow \alpha = \frac{4.71}{1.6}$

$$\therefore \alpha = 2.94 \text{ rad s}^{-2}$$