Atomic Spectra

PROBLEMS.

P. 20.1:- Given that: Hydrogen atom: $n = 1$.

(a) $r_1 = ?$ (b) $P = ?$ (c) $L = ?$ (d) $K.E = ?$

(e) $P.E. = ?$ (f) Total energy 'E' = ?

Sol.:- (a) Since $r_n = \dfrac{n^2 h^2}{4\pi^2 m k e^2}$ and $n = 1$

$$\therefore \quad r_1 = \frac{(1)^2 \times (6.63\times10^{-34})^2}{4\times(3.14)^2\times 9.11\times10^{-31}\times 9\times10^{9}\times(1.6\times10^{-19})^2}$$

$$\text{or} \quad r_1 = \frac{4.395\times10^{-67}}{8.277\times10^{-57}} = 0.53\times10^{-10}\ m$$

$$\therefore \quad r_1 = 0.53\times10^{-10}\ m$$

(b) $P_n = m v_n = m \times \dfrac{2\pi k e^2}{nh}$

$\therefore$ for $n = 1$, $P_1 = \dfrac{2\pi m k e^2}{h}$

$$\text{or} \quad P_1 = \frac{2\times3.14\times9.11\times10^{-31}\times9\times10^{9}\times(1.6\times10^{-19})^2}{6.63\times10^{-34}}$$

$$= \frac{1.32\times10^{-57}}{6.63\times10^{-34}} = 1.99\times10^{-24}\ kg\ m/s$$

(c) $L_n = \dfrac{nh}{2\pi}$ $\quad\therefore\quad L_1 = \dfrac{(1)\times6.63\times10^{-34}}{2\times3.14} = 1.055\times10^{-34}$

$$\therefore \quad L_1 = 1.055\times10^{-34}\ kg\ m^2/s$$

(d) $K.E_n = \dfrac{1}{2} m v_n^2 = \dfrac{1}{2}\left(\dfrac{ke^2}{r_n}\right)$, so for $n = 1$

$$K.E_1 = \frac{1}{2}\times\frac{ke^2}{r_1} = \frac{1}{2}\times\frac{9\times10^{9}\times(1.6\times10^{-19})^2}{0.53\times10^{-10}} = \frac{1.15\times10^{-28}}{0.53\times10^{-10}}$$

$$\text{or} \quad K.E_1 = \frac{2.17\times10^{-18}}{1.6\times10^{-19}}\ J = 13.58\ eV, \quad \therefore K.E_1 = 13.6\ eV$$

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(e) $P.E_n = -\frac{Ke^2}{r_n}$, for $n=1$, $P.E_1 = -\frac{Ke^2}{r_1}$

so, $P.E_1 = -\frac{9\times10^{9}\times(1.6\times10^{-19})^2}{0.53\times10^{-10}} = -4.35\times10^{-18}J = \frac{-4.35\times10^{-18}}{1.6\times10^{-19}} = -27.2\,eV$

$\therefore\ P.E_1 = -27.2\ eV.$

(f) $T.E = K.E + P.E = 13.6 + (-27.2) = -13.6$

$\therefore\ T.E = -13.6\ eV.$

P 20.2:- Given that: $\lambda = 400, 500$ and $700\,nm$.

'Energies in eV' = ?

Sol:- $\because$ the energy of a quanta is: $E = hf$

or $E = \frac{hc}{\lambda}$ $(\because c = f\lambda$ or $f = c/\lambda)$

(i) $E_1 = \frac{hc}{\lambda_1} = \frac{6.63\times10^{-34}\times10^{8}\times3}{400\times10^{-9}} = 4.97\times10^{-19}J \div 1.6\times10^{-19}$

$\therefore\ E_1 = 3.10\ eV.$

(ii) Similarly: $E_2 = \frac{hc}{\lambda_2} = \frac{6.63\times10^{-34}\times3\times10^{8}}{500\times10^{-9}} = \frac{3.98\times10^{-19}}{1.6\times10^{-19}}J = 2.49\,eV$

$\therefore\ E_2 = 2.49\ eV.$

(iii) $E_3 = \frac{hc}{\lambda_3} = \frac{6.63\times10^{-34}\times3\times10^{8}}{700\times10^{-9}} = \frac{2.84\times10^{-19}}{1.6\times10^{-19}}J = 1.77\ eV.$

$\therefore\ E_3 = 1.77\ eV.$

P 20.3:- As given: $E_i = -3.5\times10^{-19}J$, $E_f = -1.2\times10^{-18}J$

$\lambda = ?$

Sol. $\because E = hf = E_f - E_i$

$\frac{hc}{\lambda} = E_f - E_i$ or $\lambda = \frac{hc}{E_f - E_i}$

putting values: $\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{-1.2\times10^{-18}-(-3.5\times10^{-19})} = \frac{1.989\times10^{-25}}{-8.5\times10^{-19}}$

$\lambda = -2.34\times10^{-7} = 234\times10^{-9} = 234\,nm$

'−' sign is omitted

$\therefore\ \lambda = 234\ nm.$ $(\because$ length is never −ve$)$

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P. 20.4 :- As given : $n = 6$ to $n = 3$

$\because n > p \quad \therefore n = 6, \; p = 3, \; \lambda = ?$

Sol. :- Using formula : $\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$

Putting values:

$$\frac{1}{\lambda} = 1.0974 \times 10^{7}\left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$
$$= 1.0974 \times 10^{7}\left(\frac{1}{9} - \frac{1}{36}\right)$$
$$= 1.0974 \times 10^{7}\left(\frac{4-1}{36}\right)$$
$$= \frac{3.29 \times 10^{7}}{36} = 9.145 \times 10^{5}$$

or $\lambda = 1.09349 \times 10^{-6}\, m = 1.094 \times 10^{-6}\, m$

$\therefore \lambda = 1094\, nm.$

P. 20.5 :- For Balmer series : $p = 2$,

Sol. :- For shortest wavelength : $n = \infty$

$\therefore$ Using formula : $\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$

Putting values: $\frac{1}{\lambda} = 1.0974 \times 10^{7}\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$$= 1.0974 \times 10^{7}\left(\frac{1}{4}\right) = 0.27435 \times 10^{7}$$

or $\lambda = 3.6449 \times 10^{-7} = 364.5 \times 10^{-9} = 364.5\, nm$

$\therefore \lambda = 364.5\, nm, \quad n = \infty.$

P. 20.6 :- For Paschen series : $p = 3$,

Sol. :- For longest wavelength : $n = p + 1 = 3 + 1 = 4$

Using formula : $\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$

Putting values: $\frac{1}{\lambda} = 1.0974 \times 10^{7}\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$$= 1.0974 \times 10^{7}\left(\frac{1}{9} - \frac{1}{16}\right)$$
$$= 1.0974 \times 10^{7}\left(\frac{16-9}{144}\right)$$
$$= 5.33 \times 10^{5}$$

$\therefore \lambda = 1.875 \times 10^{-6}\, m = 1875 \times 10^{-9}\, m = 1875\, nm.$

P. 20.7 :- As given: $V = 3000$ volts, $\lambda_{min} = ?$

Sol. :- As: $K.E_{max} = hf_{max}$

and: $K.E_{max} = Ve \Rightarrow hf_{max} = Ve$

or $\frac{hc}{\lambda_{min}} = Ve$ (for f_{max}, wavelength 'λ_{min}')

or $\lambda_{min} = \frac{hc}{Ve} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3000\times1.6\times10^{-19}} = 4.14\times10^{-10}$ m

$\therefore \lambda_{min} = 4.14\times10^{-10}$ m.

P. 20.8 :- As given: $\lambda = 1.377\times10^{-10}$ m, $\Delta E = ?$

Sol. :- Using formula: $\Delta E = hf = \frac{hc}{\lambda}$

Putting values: $\Delta E = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.377\times10^{-10}} = 1.44\times10^{-15}$ J

or $\Delta E = \frac{1.44\times10^{-15}}{1.6\times10^{-19}}$ J $= 9027.7$ ev $= 9.03\times10^{3}$ ev

$\therefore \Delta E = 9.03$ Kev.

P. 20.9 :- As Pot. diff $= V = 40$ KV $= 40{,}000$ volts, $\lambda_{min} = ?$

Sol. :- Since $Ve = hf_{max} = \frac{hc}{\lambda_{min}}$

$\Rightarrow \lambda_{min} = \frac{hc}{Ve} = \frac{6.63\times10^{-34}\times3\times10^{8}}{40000\times1.6\times10^{-19}} = 3.1\times10^{-11}$ m

$\therefore \lambda_{min} = 0.31\times10^{-10}$ m.

P. 20.10 :- As given: $v_n = 5.456\times10^{5}$ m/s

(a) $n = ?$ (b) $r_n = ?$ (c) $E_n = ?$

Sol. :- (a) We know that:- $v_n = \frac{2\pi ke^2}{nh} \Rightarrow n = \frac{2\pi ke^2}{v_n h}$

Putting values: $n = \frac{2\times3.14\times9\times10^{9}\times(1.6\times10^{-19})^2}{5.456\times10^{5}\times6.63\times10^{-34}} = 3.999 = 4$

$\therefore n = 4$.

(b) $\because r_n = \frac{n^2h^2}{4\pi^2mke^2}$, so: $r_4 = \frac{(4)^2\times(6.63\times10^{-34})^2}{4\times(3.14)^2\times9.11\times10^{-31}\times9\times10^{9}\times(1.6\times10^{-19})^2}$

or $r_4 = 0.849\times10^{-9}$ m $= 0.85\times10^{-9}$ m, $\therefore r_4 = 0.85$ nm.

(c) $E_n = -\frac{2\pi^2mk^2e^4}{n^2h^2}$, so: $E_4 = -\frac{2\times(3.14)^2\times9.11\times10^{-31}\times(9\times10^{9})^2\times(1.6\times10^{-19})^4}{(4)^2\times(6.63\times10^{-34})^2}$

or $E_4 = -\frac{9.54\times10^{-85}}{7.033\times10^{-66}} = -\frac{1.36\times10^{-19}}{1.6\times10^{-19}}$ J $= -0.8478$ eV, $\therefore E_4 = -0.85$ eV.