PROBLEMS

P.15.1: An emf of 0.45V is induced between the ends of a metal bar moving through a magnetic field of 0.22T. What field strength would be needed to produce an emf of 1.5V between the ends of bar, assuming that all other factors remain the same?

DATA. 1st induced emf $= \mathcal{E}_1 = 0.45V$

Magnetic field $= B_1 = 0.22T$

2nd induced emf $= \mathcal{E}_2 = 1.5V$

Magnetic field strength $= B_2 = ?$

Sol. We know that

$$\mathcal{E}_1 = B_1 VL \sin\theta \quad \text{--- (1)}$$

and

$$\mathcal{E}_2 = B_2 VL \sin\theta \quad \text{--- (2)}$$

According to given condition, V, L and θ do not change, so dividing eq. (1) by eq. (2), we have

$$\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{B_1}{B_2} \quad \text{or} \quad B_2 = \frac{\mathcal{E}_2 B_1}{\mathcal{E}_1}$$

$$B_2 = \frac{1.5V \times 0.22T}{0.45V} = \boxed{0.73T}$$

(P.T.O)

P.15.2: The flux density B in a region between the pole faces of a horse shoe magnet is $0.5\ Wb\,m^{-2}$ directed vertically downward. Find the emf induced in a straight wire 5.0 cm long, perpendicular to B when it is moved in a direction at an angle of $60°$ with the horizontal with a speed of $100\ cm\,s^{-1}$.

DATA. Flux density $= B = 0.5\ Wb\,m^{-2}$ (vertically downward)

Length of wire $= L = 5.0\ cm = 0.05\ m$

$\theta = 60°$ with horizontal $= 30°$ with B (vertical)

speed of wire $= v = 100\ cm\,s^{-1} = 1.0\ m\,s^{-1}$

emf induced $= \mathcal{E} = ?$

Sol. As $\mathcal{E} = vBL\sin\theta$

$= 1.0\ m\,s^{-1} \times 0.5\ Wb\,m^{-2} \times 0.05\ m \times \sin 30°$

$= \boxed{1.25 \times 10^{-2}\ V}$

P.15.3: A coil of wire has 10 loops. Each loop has an area of $1.5\times10^{-3}\ m^2$. A magnetic field is perpendicular to the surface of each loop at all times. If the magnetic field is changed from 0.05 T to 0.06 T in 0.1 s, find the average emf induced in the coil during this time.

DATA. No. of loops of a coil $= N = 10$

Area of each loop $= A = 1.5\times10^{-3}\ m^2$

change in magnetic field $= \Delta B = 0.06T - 0.05T = 0.01T$

Time interval $= \Delta t = 0.1\ s$

emf induced $= \mathcal{E} = ?$

Sol. The magnitude of the induced emf in the coil is;

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t} \quad (1)$$

As $\Delta\phi = \Delta B\, A$

$$\therefore \quad \mathcal{E} = N\frac{(\Delta B)A}{\Delta t} \quad (2)$$

Putting the values, we have $\qquad \because T = N A^{-1} m^{-1}$

$$\mathcal{E} = \frac{10 \times 0.01\ T \times 1.5\times10^{-3}\ m^2}{0.1\ s}$$

$$\mathcal{E} = \boxed{1.5\times10^{-3}\ V}$$

(P.T.O)

P.15.4: A circular coil has 15 turns of radius 2 cm each. The plane of the coil lies at $40°$ to a uniform magnetic field of 0.2T. If the field is increased to 0.5 T in 0.2 s, find the magnitude of the induced emf.

DATA. Number of turns $= N = 15$

Radius of the coil $= r = 2\,cm = 2\times10^{-2}\,m$

Angle b/w $\vec{B}$ and the plane of coil $= \theta = 40°$

Increase in magnetic field $= B = 0.5T - 0.2T = 0.3T$

Time interval $= \Delta t = 0.2\,S$

Induced emf $= \mathcal{E} = ?$

Sol. The magnitude of induced emf is given by;

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t} \quad \text{(1)}$$

But $\Delta\phi = \Delta\vec{B}\cdot\vec{A} = \Delta B(\pi r^2)\cos 40°$ $(\because A = \pi r^2)$

$$\therefore \quad \mathcal{E} = \frac{N(\Delta B)(\pi r^2)\cos 40°}{\Delta t} \quad \text{(2)}$$

Putting the values, we have;

$$\mathcal{E} = \frac{15\times0.3\,T\times3.14\times(0.02\,m)^2\times0.766}{0.2\,S} \quad (\because \cos 40° = 0.766)$$

$$\mathcal{E} = \frac{4.33\times10^{-3}}{0.2} = \boxed{0.021\ V}$$

P.15.5. Two coils are placed side by side. An emf of 0.8 V is observed in one coil when the current is changing at the rate of $200\,A\,s^{-1}$ in the other coil. What is the mutual inductance of the coils?

DATA. Emf induced in the one coil $= \mathcal{E}_s = 0.8\,V$

Rate of change of current $= \frac{\Delta I_P}{\Delta t} = 200\,A\,s^{-1}$

Mutual inductance of the coil $= M = ?$

Sol. As $\mathcal{E}_s = M\frac{\Delta I_P}{\Delta t}$

$$M = \frac{\mathcal{E}_s}{\Delta I_P/\Delta t} = \frac{0.8\,V}{200\,A\,s^{-1}}$$

$$M = \frac{8}{2000} = 4\times10^{-3}\,H = \boxed{4\,mH}$$

(P.T.O)

P.15.6: A pair of adjacent coils has a mutual inductance of 0.75 H. If the current in the primary changes from 0 to 10 A in 0.025 S, what is the average induced emf in the secondary? What is the change in flux in it if the secondary has 500 turns?

DATA. Mutual inductance $= M = 0.75\,H$

change of current in primary coil $= \Delta I = 10A - 0A = 10A$

Time $= \Delta t = 0.025\,S$

No. of turns of secondary $= N_s = 500$

Average induced emf in secondary coil $= \mathcal{E}_s = ?$

change of flux in secondary coil $= \Delta\phi = ?$

Sol. The emf produced in secondary, when current is changed through primary is given by;

$$\mathcal{E}_s = M\frac{\Delta I_p}{\Delta t} \quad \text{--- (1)}$$

Putting the values, we have;

$$\mathcal{E}_s = 0.75\,H \times \frac{10\,A}{0.025\,S}$$

$$\mathcal{E}_s = \boxed{300\,V} \quad \text{--- (2)}$$

The emf produced in secondary can also be expressed as;

$$\mathcal{E}_s = N_s\frac{\Delta\phi_s}{\Delta t}$$

$$\Delta\phi = \frac{\mathcal{E}_s \times \Delta t}{N_s} = \frac{300\,V \times 0.025\,S}{500} = \boxed{1.5\times10^{-2}\,Wb}$$

P.15.7: A solenoid has 250 turns and its self inductance is 2.4 mH. What is the flux through each turn when the current is 2A? What is the induced emf when the current changes at 20 A s^{-1}?

DATA. No. of turns $= N = 250$

Self inductance $= L = 2.4\,mH = 2.4\times10^{-3}\,H$

Current $= I = 2A$

Rate of change of current $= \frac{\Delta I}{\Delta t} = 20\,A\,s^{-1}$

Flux through each turn $= \phi = ?$

Induced emf $= \mathcal{E} = ?$

(P.T.O.)

Sol. The magnitude of self induced emf is given by;

$$\varepsilon = L\frac{\Delta I}{\Delta t} \quad \text{(1)}$$

Putting values, we get;

$$\varepsilon = 2.4\times10^{-3}\,H \times 20\,A\,s^{-1}$$

$$\varepsilon = 48.0\times10^{-3}\,V \quad \text{(2)}$$

$$\varepsilon = \boxed{48.0\,mV}$$

From Faraday's law, we know that

$$\varepsilon = N\frac{\Delta \phi}{\Delta t} \quad \text{(3)}$$

Comparing eq. (1) and (3), we have;

$$N\frac{\Delta \phi}{\Delta t} = L\frac{\Delta I}{\Delta t}$$

$$\Delta\phi = \frac{L}{N}\times \Delta I$$

When **constant current** passes through the coil, then

$$\phi = \frac{L}{N}\times I \quad \text{(4)}$$

Putting the values, we have

$$\phi = \frac{2.4\times10^{-3}\,H \times 2A}{250} = \boxed{1.92\times10^{-5}\,Wb}$$

P.15.8: A solenoid of length 8.0 cm and cross sectional area 0.5 cm² has 520 turns. Find the self inductance of the solenoid when the core is air. If the current in the solenoid increases through 1.5A in 0.2S, find the magnitude of induced emf in it.

DATA. Length of solenoid $= l = 8.0\,cm = 0.08\,m$

Cross sectional area of solenoid $= A = 0.5\,cm^2 = 0.5\times10^{-4}\,m^2$

No. of turns $= N = 520$

No. of turns per unit length $= n = \frac{N}{l} = \frac{520}{0.08}\,m^{-1}$

Increase in current $= \Delta I = 1.5\,A$

Time for increase $= \Delta t = 0.2\,S$

Permeability of air $= \mu_o = 4\pi\times10^{-7}\,Wb\,A^{-1}\,m^{-1}$

Self inductance for air cored solenoid $= L = ?$

Induced emf $= \varepsilon = ?$

Sol.

(P.T.O)

The self inductance of air cored solenoid is given by;

$$L = \mu_o n^2 l A \quad \text{---} (1)$$

Putting the values, we have

$$L = 4\pi \times 10^{-7}\, Wb\, A^{-1} m^{-1} \times \left(\frac{520}{0.08}\right)^2 m^{-2} \times 0.08\, m \times 0.5 \times 10^{-4}\, m^2$$

$$L = \boxed{2.12 \times 10^{-4} H} \quad \text{---} (2)$$

The self induced emf is given by;

$$\mathcal{E} = L\frac{\Delta I}{\Delta t} \quad \text{---} (3)$$

Putting the values, we get

$$\mathcal{E} = 2.12 \times 10^{-4} H \times \frac{1.5}{0.2} A s^{-1}$$

$$\mathcal{E} = \boxed{1.6 \times 10^{-3} V}$$

P.15.9: When current through a coil changes from 100 mA to 200 mA in 0.005 s, an induced emf of 40 mV is produced in the coil (a) What is the self inductance of the coil? (b) Find the increase in the energy stored in the coil.

DATA. $I_1 = 100\, mA = 0.1\, A$, $I_2 = 200\, mA = 0.2\, A$

change in current $= \Delta I = 200\, mA - 100\, mA = 100\, mA = 100 \times 10^{-3} A$

Time $= \Delta t = 0.005\, s$

Induced emf $= \mathcal{E} = 40\, mV = 40 \times 10^{-3} V$

(a) Self inductance of the coil $= L = ?$

(b) Increase in energy stored in the coil $= \Delta U_m = ?$

Sol.(a) The self induced emf in a coil is given by;

$$\mathcal{E} = L\frac{\Delta I}{\Delta t} \quad \text{---} (1)$$

$$\text{or} \quad L = \frac{\mathcal{E}}{\Delta I/\Delta t} = \frac{40 \times 10^{-3} V}{(0.1 A/0.005 s)} = 2 \times 10^{-3} H = \boxed{2\, mH}$$

(b) If $(U_m)_i$ is initial and $(U_m)_f$ is the final energy stored in the coil, then change in energy stored is given by;

$$\Delta U_m = (U_m)_f - (U_m)_i = \frac{1}{2}LI_2^2 - \frac{1}{2}LI_1^2$$

$$\Delta U_m = \frac{1}{2}L(I_2^2 - I_1^2) \quad \text{---} (2)$$

Putting the values, we get;

$$\Delta U_m = \frac{1}{2} \times 2 \times 10^{-3} H\{(0.2 A)^2 - (0.1 A)^2\}$$

$$\Delta U_m = 0.03 \times 10^{-3} J = \boxed{0.03\, mJ}$$

(P.T.O)

15-53

P.15.10: Like any field, the earth's magnetic field stores energy. Find the magnetic energy stored in a space where strength of earth's field is 7×10^{-5} T, if the space occupies an area of $10\times10^{8}\ m^2$ and has a height of 750 m.

DATA. Strength of magnetic field $= B = 7\times10^{-5}$ T

Area $= A = 10\times10^{8}\ m^2$

Height of area above the earth surface $= h = \ell = 750$ m

Permeability $= \mu_o = 4\pi\times10^{-7}\ Wb\,A^{-1}m^{-1}$

Magnetic energy stored in space $= U_m = ?$

Sol. The magnetic energy density is

$$U_m' = \frac{B^2}{2\mu_o} \quad (1)$$

$$= \frac{(7\times10^{-5}\ T)^2}{2\times4\pi\times10^{-7}\ Wb\,A^{-1}m^{-1}} = 1.95\times10^{-3}\ J\,m^{-3} \quad (2)$$

The magnetic energy stored is :

$$U_m = U_m' \times \text{Volume} \quad (3)$$

$$U_m = U_m' \times A\ell$$

$$= 1.95\times10^{-3}\ J\,m^{-3}\times10\times10^{8}\ m^2\times750\ m$$

$$U_m = \boxed{1.46\times10^{9}\ J}$$

P.15.11: A Square coil of side 16 cm has 200 turns and rotates in a uniform magnetic field of magnitude 0.05 T. If the peak emf is 12V, what is the angular velocity of the coil?

DATA. Area of square coil $= A = 16cm\times16cm = 256\ cm^2 = 2.56\times10^{-2}\ m^2$

No. of turns $= N = 200$

Strength of magnetic field $= B = 0.05$ T

Maximum emf $= \varepsilon_o = 12$ V

Angular velocity of the coil $= \omega = ?$

Sol. The peak value of voltage generated by A.C generator is

$$\varepsilon_o = B\omega NA$$

or $$\omega = \frac{\varepsilon_o}{BNA} = \frac{12V}{0.05T\times200\times2.56\times10^{-2}\ m^2}$$

$$\omega = 46.87\ rad\ s^{-1}$$

$$\omega \simeq \boxed{47\ rad\ s^{-1}}$$

(P.T.O)

P.15.12: A generator has a rectangular coil consisting of 360 turns. The coil rotates at 420 rev per min. in 0.14 T magnetic field. The peak value of emf produced by the generator is 50V. If the coil is 5 cm wide, find the length of the side of the coil.

DATA. No. of turns $= N = 360$

Angle of frequency $= \omega = 420 \text{ rev min}^{-1}$

$= \frac{420 \times 2\pi \text{ rad}}{60 s} = 43.96 \text{ rad s}^{-1}$ $(\because 1 \text{ rev} = 2\pi \text{ rad})$

strength of magnetic field $= B = 0.14 T$

Peak value of emf $= \mathcal{E}_o = 50V$

Width of coil $= b = 5.0 \text{ cm} = 0.05 m$

Length of coil $= l = ?$

Sol. The peak value of emf generated by generator is:

$$\mathcal{E}_o = B\omega NA \quad \text{(1)}$$

Where 'A' is the area of coil given by $A = l \times b$

so $\mathcal{E}_o = B\omega N l \times b$

or $$l = \frac{\mathcal{E}_o}{B\omega N \times b} \quad \text{(2)}$$

Putting the values, we have

$$l = \frac{50V}{0.14T \times 43.96 \text{ rad s}^{-1} \times 360 \times 0.05m}$$

$$l = \boxed{0.45m = 45 \text{ cm}}$$

P.15.13: It is desired to make an a.c generator that can produce an emf of 5KV with 50 Hz frequency. A coil of area $1m^2$ consisting of 200 turns is used as armature. What should be the magnitude of the magnetic field in which the coil rotates?

DATA. Electromotive force (emf) $= \mathcal{E}_o = 5KV = 5000V$

Frequency $= f = 50$ Hz ; No. of turns $= N = 200$

Area of coil $= A = 1 m^2$; Magnetic field $= B = ?$

Sol. As $\mathcal{E}_o = B\omega NA = BNA \times 2\pi f$ $\quad \because \omega = 2\pi f$

or $$B = \frac{\mathcal{E}_o}{2\pi f NA} = \frac{5000V}{2 \times 3.14 \times 50 Hz \times 200 \times 1 m^2}$$

$$B = \boxed{0.08 T}$$

(P.T.O)

P.15.14: The back emf in a motor is 120V when the motor is turning at 1680 rev per min. What is the back emf when the motor turns 3360 rev per min?

DATA. Back emf $= \mathcal{E}_1 = 120\text{V}$

1st value of angular frq. $= \omega_1 = 1680 \text{ rev min}^{-1}$

2nd " " " $= \omega_2 = 3360 \text{ rev min}^{-1}$

emf corresponding to $\omega_2 = \mathcal{E}_2 = ?$

Sol. As $\mathcal{E} = B\omega NA$ ——— (1)

so $\mathcal{E}_1 = B\omega_1 NA$ ——— (2)

and $\mathcal{E}_2 = B\omega_2 NA$ ——— (3)

Dividing eq. (1) by (2), we have

$$\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{\omega_1}{\omega_2} \quad \text{——— (4)}$$

i.e; the back emf produced in motor is proportional to the angular velocity of coil.

From eq. (4), we have

$$\mathcal{E}_2 = \mathcal{E}_1 \times \frac{\omega_2}{\omega_1} \quad \text{——— (5)}$$

Putting the values, we have

$$\mathcal{E}_2 = 120\text{V} \times \frac{3360 \text{ rev min}^{-1}}{1680 \text{ rev min}^{-1}} = \boxed{240\text{V}}$$

P.15.15: A D.C motor operates at 240V and has a resistance of 0.5Ω. When the motor is running at normal speed, the armature current is 15A. Find the back emf in the armature.

DATA. Operating voltage $= V = 240\text{V}$

Resistance $= R = 0.5\,\Omega$

Armature current $= I = 15\text{ A}$

Back emf $= \mathcal{E} = ?$

Sol. The back emf can be calculated as

$$V = \mathcal{E} + IR$$

or $\mathcal{E} = V - IR$ ——— (1)

Putting the values, we have

$$\mathcal{E} = 240 - 15 \times 0.5 = \boxed{232.5\text{ V}}$$

(P.T.O)

P.15.16: A copper ring has a radius of 4cm and resistance of 1.0 mΩ. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2T to 0.4T in a time interval of 5×10^{-3} s, what is the current in the ring during this interval?

DATA. Radius of copper ring $= r = 4.0\text{ cm} = 0.04\text{ m}$

Resistance of copper ring $= R = 1\text{ m}\Omega = 1\times10^{-3}\,\Omega$

Increase in magnetic field $= \Delta B = B_2 - B_1 = 0.4T - 0.2T = 0.2T$

Time for increase $= \Delta t = 5\times10^{-3}$ s

current in ring during this interval $= I = ?$

Sol. The emf induced in the ring is given by Faraday's law;

$$\varepsilon = N\frac{\Delta\phi}{\Delta t} \quad \text{(1)}$$

Here $N=1$ and $\Delta\phi = \Delta BA = \Delta B\times\pi r^2$

$$\therefore \quad \varepsilon = N\times\frac{\Delta B\times\pi r^2}{\Delta t} \quad \text{(2)}$$

Putting the values, we have;

$$\varepsilon = 1\times\frac{0.2T\times3.14\times(0.04)^2m^2}{5\times10^{-3}s} = 0.20096\text{V}$$

$$\varepsilon = \boxed{2.0\times10^{-1}\text{V}} \quad \text{(3)}$$

From Ohm's law, the current through ring is given by;

$$I = \frac{\varepsilon}{R}$$

$$I = \frac{2.0\times10^{-1}\text{V}}{1\times10^{-3}\,\Omega} = 200.96\text{ A} \simeq \boxed{201\text{A}}$$

P.15.17: A coil of 10 turns and 35 cm^2 area is in a perpendicular magnetic field of 0.5T. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field.

DATA. No. of turns $= N = 10$

Area $= \Delta A = 35\text{ cm}^2 = 35\times10^{-4}\text{ m}^2$

Magnetic field $= B = 0.5T$

Time $= \Delta t = 1.0$ s

Induced emf $= \varepsilon = ?$

Sol. The magnitude of induced emf by Faraday's law is;

$$\varepsilon = N\frac{\Delta\phi}{\Delta t} = N\times\frac{B\Delta A}{\Delta t} \quad (\because \Delta\phi = B\Delta A)$$

$$\varepsilon = 10\times\frac{0.5T\times35\times10^{-4}m^2}{1.0\text{ s}} = \boxed{1.75\times10^{-2}\text{V}}$$

(P.T.O)

P.15.18: An ideal transformer (step down) is connected to main supply of 240V. It is desired to operate a 12V, 30W lamp. Find the current in the primary and the transformation ratio?

DATA. Primary voltage $= V_P = 240\,V$

Secondary voltage $= V_S = 12\,V$

output power $= P_S = 30\,W$

Current in primary $= I_P = ?$

Transformation ratio $= \frac{N_S}{N_P} = ?$

Sol. As input power = output power

$$P_P = P_S$$

or,
$$V_P I_P = P_S$$

or,
$$I_P = \frac{P_S}{V_P} = \frac{30\,W}{240\,V} = \boxed{0.125\,A}$$

Transformation ratio is:

$$\frac{N_S}{N_P} = \frac{V_S}{V_P}$$

$$\frac{N_S}{N_P} = \frac{12\,V}{240\,V} = \boxed{\frac{1}{20}}$$

END