CHAPTER 19

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Dawn Of Modern Physics

Before 1890 A.D. — classical mechanics

Classical Physics → classical physics, Newtonian physics / Newtonian mechanics

Def:- The physics up to 1890 A.D. was termed as Classical Physics. It is based upon Newton's laws. This physics deals with the study of macroscopic (large) objects (i.e. bodies of larger masses but of smaller velocities). This is still valid in ordinary processes of everyday life. ⇒ related with macroscopic bodies moving with small speed.

Failure of Classical Physics:-

Attempts to explain the behaviour of matter on the atomic level with the laws of classical physics were not successful. Phenomena such as blackbody radiation, the photoelectric effect, the emission of sharp spectral lines by atoms in a gas discharge tube and invariance of speed of light, could not be understood within the frameworks of classical physics.

Modern Physics

After 1890 A.D. to today → quantum mechanics

Def:- The physics from 1890 A.D. to onward is termed as Modern Physics.

This physics deals with the study of microscopic objects (i.e. bodies of smaller masses but of greater velocities). ⇒ small bodies moving with large speed

Importance of Modern Physics:-

The two most significant features of modern physics are relativity and quantum theory. The observations on objects moving very fast, approaching the speed of light, are well explained by the special theory of relativity (a branch of modern physics). Quantum theory has been

speed of light

$= 3\times10^8$ m/s

$= 3\times10^5\times10^3$ m/s

$= 3\times10^5$ km/s

Tiger 70 km/hr

bullet japan range 650 km/hr, 1050 km/hr

fuel made on rocket + jet engine design = 1700 km/hr

comparable

$\frac{1}{5th}$ to speed of light

⇒ relativity

L.Y. = $S = ct$ unit of distance

$= 3\times10^8 \frac{m}{s} \times 365\times24\times60\times60 s$

=

m.

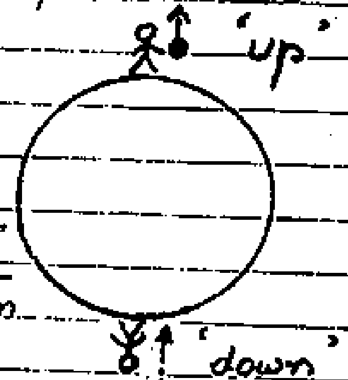
able to explain the behaviour of electromagnetic radiations as discrete packets of energy and the particles on a very small scale are dominated by wave properties.

§ 19.1 RELATIVE MOTION

1- Introduction:

(a)- Classical View :- In classical physics Galileo and Newton both believed in absolute space (absolute position) and absolute Time. Every object was considered to be either in absolute motion or at absolute rest position. Even, the direction of motion was also considered as absolute for all types of the observers.

(b)- Modern View :- Modern physics contradicts these concepts. When we say a ball is thrown up, the 'up' direction is only for that particular place. It will be 'down' position for a person on the diametrically opposite side of the globe. Therefore



"The concept of direction is purely relative"

Similarly, the rest position or the motion of an object is not same for different observers. For example, the walls of the cabin of a moving train are stationary with respect to the passengers sitting inside it but are in motion to a person stationary on the ground. So we cannot say whether an object is absolutely at rest or absolutely in motion.

"All motions are relative to a person or instrument observing it."

2- Definition

The motion of an object with respect to a certain reference point or an observer is known as relative motion.

Absolute :- motion & rest are not absolute.

Relative : Motion and rest are relative

Every thing of the world will come in relative but almighty God will come in absolute only.

(3) Explanation :-

(a) **Experimental Verification :-** Let us perform an experiment in two cars moving with constant velocities in any direction. Suppose a ball is thrown straight up. It will come back straight down. This will happen in both cars. Suppose now one car is stationary. The person in the other car, which is moving with constant velocity, throws a ball straight up. He will receive the ball straight down. On the other hand, the fellow sitting in the stationary car observes that the path of the ball is not a straight line but a parabola.

(b). **Conclusions:-** Hence it is concluded that

(i) The observers in their own frame of reference have identical observations in all respects, but if they look into other frames, they observe differently.

(ii) Modern physics gave a new idea about relative motion and rest. There is nothing like absolute rest or absolute motion. So in our daily life, the motion or rest we talk about, the velocity or acceleration we measure are all relative.

§ 19.2 FRAME OF REFERENCE

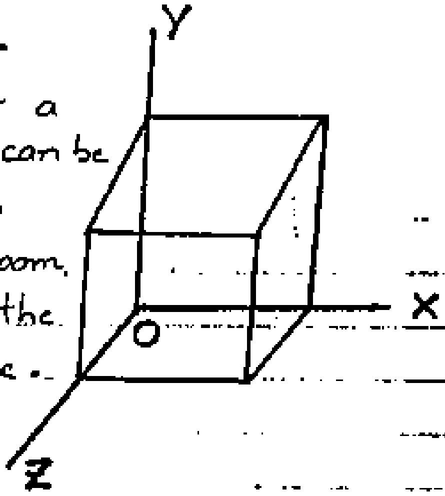
1. **Definition :-** A frame of reference is any coordinate system relative to which measurements are taken. So a frame of reference is required to describe the position of an object in space at a certain time.

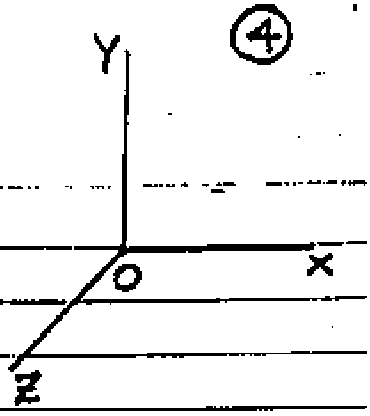
2. **Explanation :-**

(a). The simplest frame of reference is the Cartesian coordinate system which consists of three mutually perpendicular lines such as OX, OY and OZ from a common origin O as shown in figure.

(b). **Examples :-**

(i)- The position of a table in a room can be located relative to the walls of the room. The room is then the frame of reference.





(ii)- In order to make the measurements in the laboratory the laboratory is the frame of referense. If the same experiment is performed in a moving train, The train becomes a frame of reference.

(iii)- The position of a spaceship can be described relative to the positions of the distant stars. A coordinate system based on these stars is then the frame of reference.

(c). **Types :-** There are two types of frames of reference.

(i). **Inertial Frame Of Reference :-**

Def :- An inertial frame of referense is defined as a coordinate system in which the law of inertia is valid.

OR A frame of reference moving with uniform velocity w.r.t the other frame of reference or at rest is called an inertial frame. It is also known as non-accelerated frame of reference.

Explanation :- Other laws of nature also apply in such a system. If we place a body upon Earth, it remains at rest unless an unbalanced force is applied upon it. This observation shows that Earth may be considered as an inertial frame of reference.

A body placed in a car moving with a uniform velocity w.r.t Earth also remains at rest w.r.t car, so that car is also an inertial frame of reference. Thus any frame of reference which is moving with uniform velocity relative to an inertial frame is also an inertial frame.

(ii). Non-Inertial Frame Of Reference:

Def:- A non-inertial frame of reference is defined as a coordinate system in which law of inertia is not valid.

OR If a frame of reference is moving with a variable velocity (certain acceleration) w.r.to other frame of reference is called non-inertial frame or accelerated frame of reference.

Explanation:- When a moving car is suddenly stopped, the body placed in it, no longer remains at rest. So is the case when the car is suddenly accelerated. In such a situation, the car is not an inertial frame of reference.

Thus an accelerated frame is a non-inertial frame of reference.

NOTE

In nature a perfect inertial frame of reference does not exist. Since Earth is rotating and revolving hence strictly speaking the Earth is not an inertial frame. But it can often be treated as an inertial frame without serious error because of very small acceleration.

§ 19.3 SPECIAL THEORY OF RELATIVITY

Theory Of Relativity

Theory of relativity deals with the effect of relative motion on physical phenomena. These effects become more appreciable only at speeds equal to or greater than one-tenth of speed of light ($v \simeq \frac{1}{10}(c)$) called Relativistic Speed.

It was proposed by Albert Einstein.

There are two types of theory of relativity

1. General Theory of Relativity
2. Special theory of Relativity

(1) GENERAL THEORY OF RELATIVITY

(a). **Introduction**:- Albert Einstein put forward his general theory of relativity in 1916.

(b). **Statement**:- It is that branch of theory of relativity which deals with non-inertial or accelerated frames of reference.

- This theory treats problems relating to frames of reference accelerating with respect to one another. In this branch gravitational field does play some part.

(2) SPECIAL THEORY OF RELATIVITY

(a). **Introduction**:- This theory was put forward by Albert Einstein in 1905.

(b). **Statement**:- This theory deals with inertial or non-accelerating frames of reference.

- In this theory gravitational field does not play any part. This theory treats problems involving inertial frames.

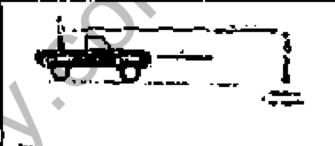
(c) **Postulates**:- The special theory of relativity is based upon two postulates which can be stated as follows:-

(i). The laws of physics are the same in all inertial frames.

- This postulate is the generalization of the fact that all physical laws are the same in frames of reference moving with uniform velocity with respect to one another. If laws of physics were different for different observers in relative motion, the observer could determine from this difference that which of them were stationary in a space and which were moving. But such a distinction does not exist, so this postulate implies that there is no way to detect absolute uniform motion.

(ii) The speed of light in free space has the same value for all observers, regardless of their state of motion.

- This postulate states an experimental fact that speed of light in free space is the universal constant $c = 3\times10^8\,ms^{-1}$.



The speed of light emitted by flashlight is c measured by two observers, one on the moving track and the other on the road.

(d). Results:-

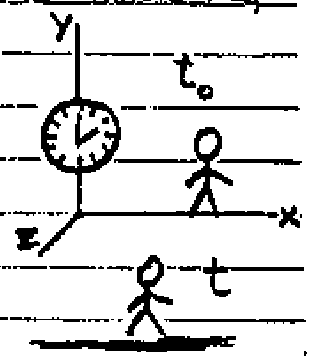
Some interesting results of the special theory of relativity can be summarized as follows without going into their mathematical derivations.

(i). Time Dilation:-

According to classical physics Time is an absolute quantity (i.e Time interval b/w two events occured in same position would be same whoever measured it.)

But according to special theory of relativity, time is not absolute quantity. It depends upon the motion of the frame of reference.

Expression:-



Suppose an observer is stationary in an inertial frame. He measures the time interval between two events in this frame. Let it be 't_o'. This is known as proper time.

If the observer is moving with respect to frame of events with very high velocity 'v' or if the frame of events is moving w. r. to observer with very high uniform velocity 'v', the time measured by the observer would not be t_0 but it would be 't' given by

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

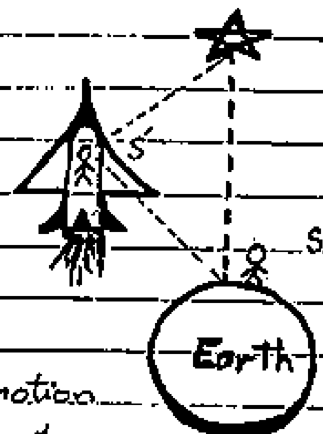
As $v \ll c$ so $\sqrt{1-v^2/c^2}$ is always less than one. Therefore t is greater than t_0.

Conclusion :- As $t > t_0$, this shows that time has dilated or stretched due to relative motion of the observer and the frame of reference of events.

Application :- This astonishing result applies to all timing processes physical, chemical and biological. Even aging process of the human body is slowed by motion at very high speeds.

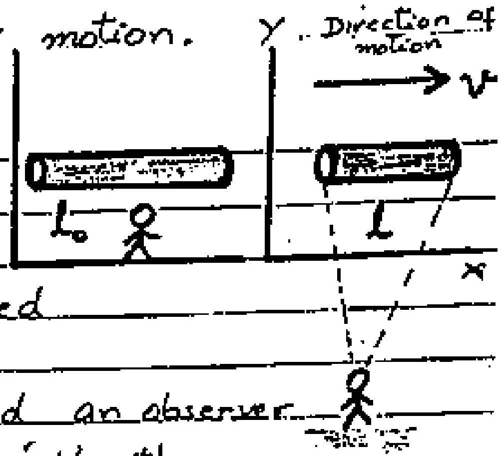
(ii). Length Contraction :-

Introduction :- The distance from Earth to a star measured by an observer in a moving spaceship would seem smaller than the distance measured by an observer on Earth. (i.e $s' < s$)



Definition :- If an observer is in motion relative to two points that are a fixed distance apart, the distance between the two points appears shorter than if the observer was at rest relative to them. This effect is known as Length Contraction.

→ The length contraction happens only along the direction of motion. No such contraction would be observed perpendicular to the direction of motion.



Expression :-

Proper length - The length of an object or distance between two points measured by an observer who is relatively at rest is called proper length l_0.

Contracted length - If an object and an observer are in relative motion with speed 'v' then the contracted length 'l' is given by

$$\boxed{l = l_0\sqrt{1-\frac{v^2}{c^2}}}$$

Conclusion :- As $v < c$, $\frac{v^2}{c^2} < 1 \Rightarrow \sqrt{1-\frac{v^2}{c^2}} < 1$

$\therefore \; l < l_0$

The length along the direction of motion has decreased.

(iii) Mass Variation :-

According to classical mechanics, mass of an object is considered as constant quantity.

But according to special theory of relativity, mass of an object is a varying quantity and depends upon the speed of the object.

Expression :-

Proper Mass - The mass of an object measured by the observer who is relatively at rest is called proper mass which is denoted by 'm_0'.

Increase in mass - An object whose mass when measured at rest is 'm_0' will have an increased mass 'm' when observed to be moving at speed 'v'.

The increased mass 'm' is given by the relation

$$\boxed{m = \frac{m_o}{\sqrt{1 - v^2/c^2}}}$$

Conclusions:- Since the term $\sqrt{1 - v^2/c^2} < 1$, so

$$m > m_o$$

- It means that mass of an object in motion increases.
- The increase in mass indicates an increase in inertia of the object at high speeds as mass is direct measure of inertia.
- No material object can be accelerated to the speed of light 'c' in free space.

Proof :- As

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

Let $v = c$

$$m = \frac{m_o}{\sqrt{1 - c^2/c^2}} = \frac{m_o}{\sqrt{1-1}} = \frac{m_o}{0}$$

$$m = \infty$$

Multiplying by 'a' on both sides

$$ma = \infty a$$

$$F = \infty$$

$F = \infty$ since $m = \infty$ because $v = c$

This shows that an infinite mass would require an infinite force to accelerate it.

Because infinite forces are not available, hence an object cannot be accelerated to the speed of light.

Importance of Einstein's Equations :-

In our everyday life, we deal with extremely small speeds, compared to the speed of light. Even the Earth's Orbital speed is only $30\ kms^{-1}$. On the other hand, the speed of light in free space is $300000\ km/s$. This is the reason why Newton's laws are valid in everyday situations. However, when experimenting with atomic particles moving with velocities approaching speed of light, the relativistic effects are very prominent and experimental results cannot be explained without taking Einstein's equations into account.

(iv) Energy - Mass Relation:-

According to special theory of relativity mass and energy are different entities but are interconvertible or we can simply say that energy can be converted into mass and mass can be converted into energy.

The total energy E and mass m of an object are related by the expression

$$\boxed{E = mc^2}$$

Where m depends on the speed of the object.

Rest Mass Energy:- At rest, the energy equivalent of an object's mass m_o is called rest mass energy E_o.

$$E_o = m_o c^2$$

As $m > m_o$ so $E > E_o$, the difference of energy $(mc^2 - m_o c^2)$ is due to motion, as such it represents the K.E of the mass. Hence

$$K.E = mc^2 - m_o c^2 = (m - m_o)c^2$$

The change in mass $(m - m_o)$ is due to change in energy ΔE or in other words. K.E of the object appears as increase in mass.

$$\boxed{\Delta m = \frac{\Delta E}{c^2}}$$

Because c^2 is a very large quantity, this implies that small change in mass require very large changes in energy.

Note :- In our everyday world, energy changes are too small to provide measurable mass changes. However, energy and mass changes in nuclear reactions are found to be exactly in accordance with the above mentioned equations.

(e) Applications of Special theory of Relativity

(NAVSTAR Navigation System)

The results of special theory of relativity are put to practical use even in everyday life by a modern system of navigation satellites called NAVSTAR (A system to determine location and speed of an object anywhere on Earth).

i — Speed :- The speed anywhere on Earth can now be determined to an accuracy of about $2\,cm\,s^{-1}$. However, if relativity effects are not taken into account, speed could not be determined any closer than about $20\,cm\,s^{-1}$.

ii — Location :- By using these results the location of an aircraft after an hour's flight can be predicted to about 5.0 m as compared to about 760 m determined by without using relativistic effects.

Example 19.1 : The period of a pendulum is measured to be 3.0 s in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of 0.95c w.r.to the pendulum?

Data : $t_o = 3.0\,s$, $v = 0.95c$

$t = ?$

Solution :

Using $t = \frac{t_o}{\sqrt{1 - v^2/c^2}}$

$$t = \frac{3.0\,s}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} = \frac{3.0\,s}{\sqrt{1-(0.95)^2}}$$

$$\boxed{t = 9.6\,s}$$

Example 19.2 :- A bar 1.0 m in length and located along x-axis moves with a speed of 0.75 c w.r.to a stationary observer. What is the length of the bar as measured by the stationary observer ?

Data:

$l_o = 1.0\,m$

$v = 0.75\,c$

$l = ?$

Solution:

Using

$$l = l_o\sqrt{1-v^2/c^2}$$

$$l = 1.0\,m \times \sqrt{1-\frac{(0.75c)^2}{c^2}} = 1\times\sqrt{1-(0.75)^2}\,m$$

$$\boxed{l = 0.66\,m}$$

Example 19.3 :- Find the mass m of a moving object with speed 0.8 c.

Data:

$v = 0.8\,c$

$m = ?$

Solution: Using

$$m = \frac{m_o}{\sqrt{1-v^2/c^2}}$$

$$m = \frac{m_o}{\sqrt{1-\frac{(0.8c)^2}{c^2}}} = \frac{m_o}{\sqrt{1-(0.8)^2}}$$

$$\boxed{m = 1.67\,m_o}$$

§ 19.4 BLACK BODY RADITION

1- Black Body:

(a)- Definition:- A body which absorbs all types of the electromagnetic radiations falling on it when cold and emits all the electromagnetic radiation when hot is called a Black Body.

(b)- Construction:- A perfect black body does not exist in nature. However for nearest approach, consider a non-reflecting object such as a solid block that has a hollow cavity within it. It has a small hole and the radiation can enter or escape only through this hole. The inside is blackened with soot to make it as good an absorber and as bad a reflector as possible.

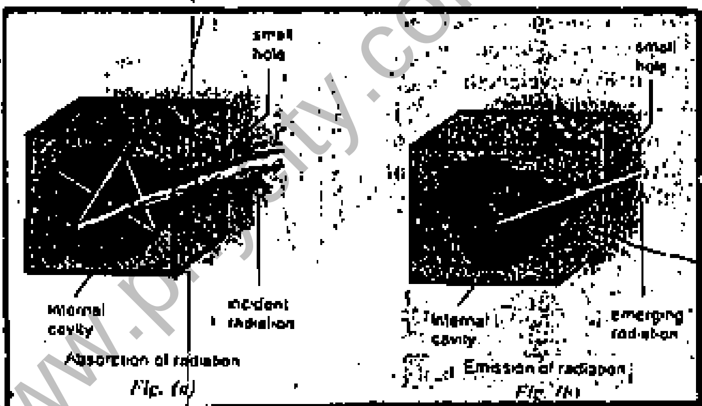


Fig. (a) Fig. (b)

The small hole appears black because the radiation that enters is reflected from inside walls many times and is partly absorbed at each reflection until none remains. Such a body is termed as black body and has the property to absorb all the radiations entering it. A black body is both an ideal absorber (as in fig (a)) and an ideal radiator (as in fig b).

(c)- Black Body Radiation:- When such a black body is heated it emits radiations of all possible wavelengths. The radiations emitted from such a body are known as Black Body Radiations, cavity radiations or temperature radiations.

(d) Dependance of the nature of emitted radiation upon temperature :-

When a body is heated, it emits radiation. The nature of radiation depends upon temperature. At low temperature, a body emits radiation which is principally of long wavelengths in invisible infrared region. At high temperature, the proportion of shorter wavelength radiation increases. Futhermore, the amount of emitted radiation is different for wavelengths.

e.g :- When platinum wire is heated, it appears dull red at about 500°C, changes to cherry red at 900°C, becomes orange red at 1100°C, yellow at 1300°C and finally white at about 1600°C.

This shows that as the temperature is increased, the radiation becomes richer in shorter wavelength (i.e wavelength decreases).

2. Intensity Distribution Diagram

(a) Introduction :- Lummer and Pringsheim measured the intensity of emitted energy with wavelength radiated from a black body at different temperatures by the apparatus shown in figure.

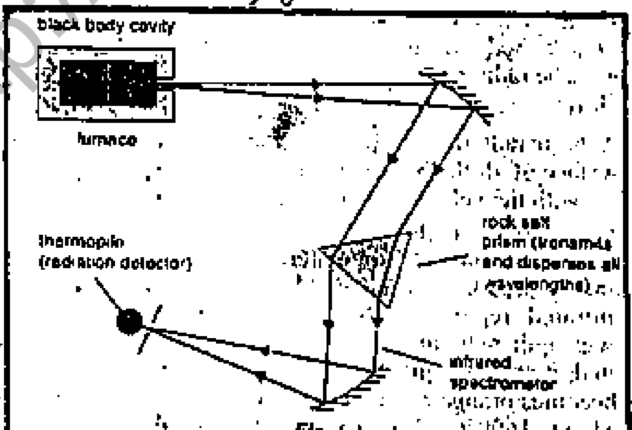


Fig. (a)

(b). Definition :- The intensity distribution diagram is actually the graphs of intensity of radiated energy against wavelength from a black body at different temperatures.
The amount of radiation emitted with different wavelengths is shown in the form of energy distribution curves for each temperature in fig (b).

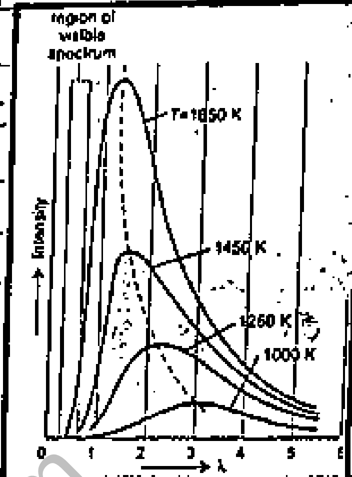


Fig (b): Results of Lummer and Pringsheim's experiments: graphs of intensity of radiated energy against wavelength from a blackbody

(C) Facts of Energy Distribution Curves

These curves reveal the following interesting facts :

(i) At a given temperature, the energy is not uniformly distributed in the radiation spectrum of the body.

(ii). Wein's Displacement Law

At a given temperature T, the emitted energy has maximum value for a certain wavelength λ_{max}.
This law states that "The wavelength corresponding to maximum intensity of emitted radiation is inversely proportional to the absolute temperature of the black body"

OR The product of wavelength corresponding to max. intensity λ_m and absolute temperature for the given curve remains constant.

$$\lambda_m \propto \frac{1}{T}$$

$$\lambda_m = \frac{\text{Constt.}}{T}$$

or $$\boxed{\lambda_m T = \text{Constt.}}$$

The value of constant is about 2.9×10^{-3} mK.
This constant is known as Wein's constt.

The above equation shows that as T increases, λ_{max} shifts to shorter wavelength.

(iii)- For all wavelengths, an increase in temperature causes an increase in energy emission.

The radiation intensity increases with increase in wavelengths and at a particular wavelength λ_{max}, it has a maximum value. With further increase in wavelength, the intensity decreases.

(iv)- The area under each curve represents the total energy radiated per second per square metre (intensity) over all wavelengths at a particular temperature.

Stephen Boltzmann Law

This law states that the area under the curve 'E' (i.e. intensity of emitted radiation) is directly proportional to the fourth power of Kelvin temperature T.

$$E \propto T^4$$

$$\boxed{E = \sigma T^4}$$

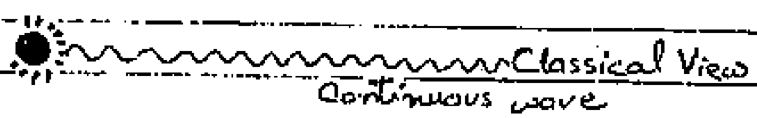
where σ is called Stephen's constant. Its value is $5.67 \times 10^{-8}\ W\, m^{-2}\, K^{-4}$.

3- Planck's Assumption

Electromagnetic wave theory of radiation cannot explain the energy distribution along the intensity wavelengths curves.

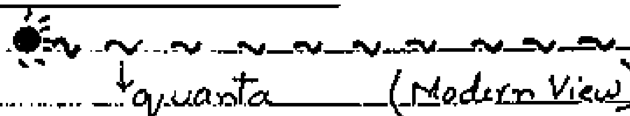
In 1900, Max Planck founded a mathematical model resulting in an equation that describes the shape of observed curves exactly (i.e. for shorter as well as longer wavelengths).

All the previous theories assumed that energy exchange between the radiation and material took place in a continuous way.



Planck's Quantum Theory

Max Planck suggested that energy is radiated or absorbed in discrete packets, called quanta rather than as a continuous wave.



Each quantum is associated with radiation of a single frequency. According to this theory "The energy E of each quantum is proportional to its frequency f."

$$E \propto f$$

$$\boxed{E = hf}$$

Where h is Planck's constant. Its value is 6.63×10^{-34} Js.

Nobel Prize: Max Planck received the Nobel Prize in physics in 1918 for his discovery of energy quanta.

4- The Photon

(a)- Max Planck suggested that radiation energy from a source is discontinuous. Just as matter is not continuous but consists of a large number of tiny particles called atoms or molecules, the same is the case with emitted energy. He assumed that granular nature of radiation from hot bodies was due to some property of the atoms producing it.

(b). Einstein's Extension

(i)- **Def. of Photon:** The emission of energy from a matter is not continuous, it is emitted in the form of packets or tiny bundles of energy which are integral part of all electromagnetic radiation and that they could not be subdivided.

These indivisible tiny bundles of energy Einstein called photons.

(ii) Energy of Photon:- The beam of light with wavelength λ consists of stream of photons travelling at speed c and carries energy $\boxed{E = hf}$

(iii) Momentum of Photon:-

From the theory of relativity momentum 'p' of photon is related to energy as $E = mc^2 = mc\,c = pc$ —— ① $(\because p = mc)$

As $E = hf$ —— ②

By comparing eq ① and ②

$$pc = hf$$

$$p = hf/c$$

or $\boxed{p = h/\lambda}$ $(\text{As } c = f\lambda)$

(iv) Rest Mass of Photon:-

The relativistic expression of mass is given as

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

This relation is only valid when m_o has some value

As photon travels with speed of light

$v = c$

$$\therefore m = \frac{m_o}{\sqrt{1 - c^2/c^2}} = \frac{m_o}{\sqrt{1-1}} = \frac{m_o}{0} = \infty$$

Hence $m = \infty$

or $mc^2 = \infty c^2$

$E = \infty$

$E = \infty$ $\therefore m = \infty$ m_o = some value

But $E \neq \infty$ $\therefore m \neq \infty$ $\therefore m_o$ has no value

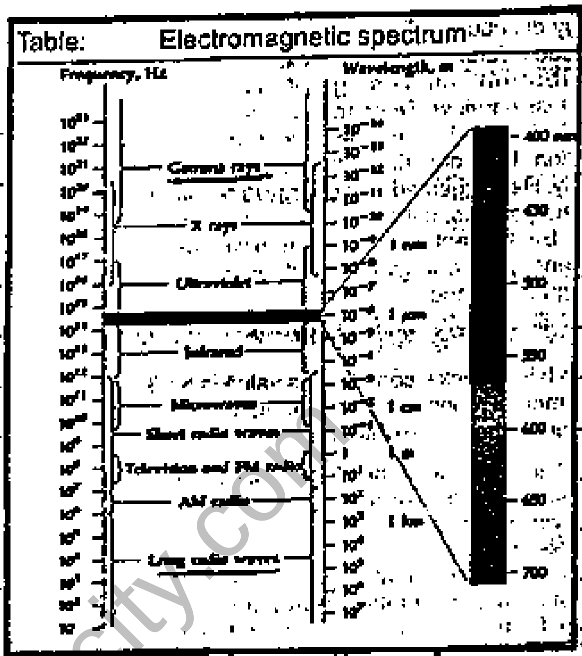
$(\because E = hf)$ As the photon does not possess infinity energy, so it is concluded that rest mass of photon must be zero i.e $\boxed{m_o = 0}$

(v). Electromagnetic Spectrum

The table shows the quanta emitted in different regions of the electromagnetic spectrum with energy.

At the high end γ-radiation with energy $\simeq 1\,MeV$ is easily detected as quanta by a radiation detector and counter.

At the other end the radio waves have energy $\simeq 10^{-11}\,eV$. So, millions of photons are needed to detect this signal. Their quanta are so close together in energy value that radio waves are detected as continuous radiations.



The emission or absorption of energy in discrete steps may be applied to any system like vibrating mass-spring system. However in this case energy steps are very small to be detected and so any granular nature is invisible. Quantum effects are only important when observing atomic sized objects where h is a significant factor in any detectable energy change.

Example 19.4:- Assuming you radiate as does a blackbody at your body temp. about 37°C, at what wavelength do you emit the most energy?

Data:- $T = 37 + 273 = 310\,K$

Wein's Constt $= 2.9 \times 10^{-3}\,mK$

$\lambda_{max} = ?$

Solution :- Using $\lambda_{max} \times T = \text{Constt.}$

$$\lambda_{max} = \frac{\text{Constt.}}{T} = \frac{2.9\times10^{-3}\,mK}{310\,K} = 9.35\times10^{-6}\,m$$

$$\lambda_{max} = 9.35\,\mu m$$

The radiation lies in the invisible infrared region and is independent of skin colour.

Example 19.5 :- What is the energy of a photon in a beam of infrared radiation of wavelength 1240 nm?

Data :- $\lambda = 1240\,nm = 1240\times10^{-9}\,m$

$E = ?$

Solution :- $E = hf = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,Js \times 3\times10^{8}\,ms^{-1}}{1240\times10^{-9}\,m}$

$E = 1.6\times10^{-19}\,J = \frac{1.6\times10^{-19}}{1.6\times10^{-19}}\,eV$

or $\boxed{E = 1.0\,eV}$

§ 19.5 INTERACTION OF ELECTROMAGNETIC RADIATION WITH MATTER

Electromagnetic radiation or photons interact with matter in three distinct ways depending mainly on their energy. The three processes are:

1. Photoelectric Effect
2. Compton Effect
3. Pair Production

(1) PHOTOELECTRIC EFFECT

(a) Definition : The emission of electrons from a metal surface when light of suitable frequency falls upon it, is called photoelectric effect.

The emitted electrons are known as photoelectrons.

(b) Explanation:-

(i) Experimental Arrangement

The apparatus used to observe the photoelectric effect is shown in fig (a). An evacuated glass tube X contains two electrodes. The electrode A connected to the positive terminal of the battery is known as anode. The electrode C connected to negative terminal is known as cathode.

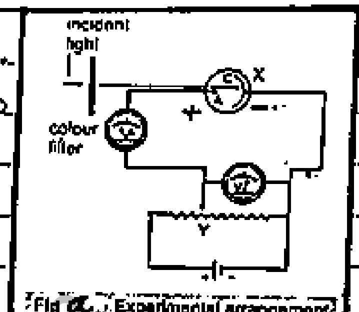


Fig a. Experimental arrangement to observe photoelectric effect.

When monochromatic light is allowed to shine on cathode, it begins to emit electrons. These photoelectrons are attracted by the positive anode and the resulting current is measured by an ammeter. When light is cut off, the current stops, which proves that the current flows because of incident light. This current is, hence, called photoelectric current.

(ii) Maximum K.E of Photoelectrons

The max. energy of the photoelectrons can be determined by reversing the connections of the battery in the circuit i.e now the anode A is negative and cathode C is at positive potential. In this condition the photoelectrons are repelled by the anode and photoelectric current decreases. If this potential is made more and more negative at a certain value, no photoelectron will reach at anode A. This potential is called stopping potential.

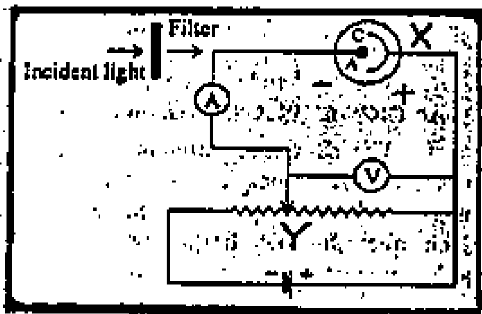


Fig (b)

V. Stopping Potential, The stopping potential is that retarding voltage for which the current becomes zero.

Even the electrons of max. energy are not able to reach collector plate.

The max. energy of photoelectrons is thus

$$(K.E)_{max} = V_0 e$$

$$\boxed{\frac{1}{2} m v_{max}^2 = V_0 e}$$

Where m is mass, v is velocity and e is the charge on electron.

(iii) Graphical Representation of Photoelectric Effect.

Case - A When frequency of the light is kept constant but intensity is varied.

If the experiment is repeated with light of higher intensity, the amount of photoelectric current increases but it is cut off for the same value of stopping potential V_0.

The fig. (c) shows two curves of photoelectric current as a function of potential V, where $I_2 > I_1$.

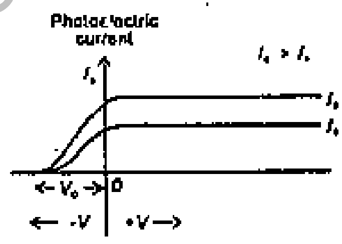


Fig C. Characteristic curves of photocurrent vs. applied voltage for two intensities of monochromatic light.

Case - B When intensity of the incident light is kept constant but frequency is varied.

If the intensity is kept constant and experiment is performed with different frequencies of incident light, we obtain the curves shown in fig (d).

The current is same but stopping potential is different for each frequency of incident light.

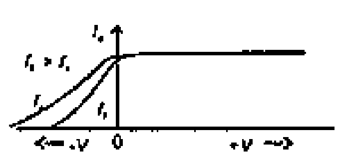


Fig d. Characteristic curves of photocurrent vs. applied voltage for light of different frequencies.

This fact shows that max. K.E of photoelectrons directly depends upon the frequency of light but independent of the intensity of light.

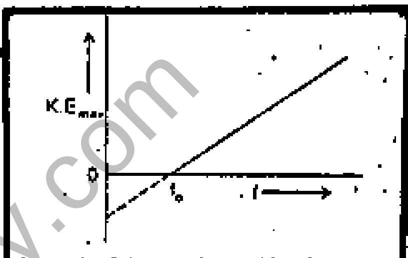
(iv). Experimental Results Of Photoelectric Effect.

The important results of the experiments are

A – The electrons are emitted with different energies. The max. energy of photoelectrons depends on the nature of metal surface and the frequency of incident light.

B – Threshold Frequency:

There is a minimum frequency below which no electrons are emitted, however intense the light may be. This minimum frequency is called threshold frequency (f_o).



A graph of the maximum kinetic energy of photoelectrons vs. light frequency. Below a certain frequency, f_o, no photoemission occurs.

Fig (e)

C – The threshold frequency depends upon the nature of the metal. It varies from metal to metal i.e different for different metals.

D – Electrons are emitted instantaneously, the intensity of light determines only their numbers (i.e the number of electrons emitted per second is directly proportional to the intensity of light).

(V). Failure Of Electromagnetic Wave Theory Of Light :-

The classical electromagnetic wave theory of light failed to explain the following facts about photoelectric effect

A – According to this theory, increasing the intensity of incident light should increase the K.E of emitted electrons which contradicts the experimental result ($\because$ K.E depends upon frequency, not intensity of light).

B – Why there should be a minimum frequency (threshold frequency) at which emission takes place?

(VI) - Explanation on the Basis of Quantum Theory

In 1905 Albert Einstein developed his explanation of photoelectric effect on the basis of quantum theory i-e quantization of energy proposed by Max Planck. According to this theory, the radiation from a source is emitted or absorbed discontinuously in packets or bundles of energy which are called quanta, known as photons. The energy of each photon of frequency 'f' as given by quantum theory is

$$E = hf$$

Einstein's Explanation:

Einstein proposed that when a beam of photons in the incident light falls on the metal surface, one photon transfers all its energy to one electron only. If the energy of photon is less than the energy required by an electron to eject from the metal surface, no photoelectron will be emitted. Increase in intensity of light means, increase in number of photons and not increase in energy. Thus, the frequency and hence energy of photon should be above some minimum value if it is to eject an electron from the metal surface with some K.E. This explain the significance of threshold frequency.

Work Function: The minimum energy, an electron must have in order to escape from the metal surface is called work function and denoted by ϕ.

Einstein Photoelectric Equation

If the energy of incident photon is sufficient, the electron is ejected instantaneously from the metal surface. A part of the photon energy (work func) is used by the electron to break away from the metal and the rest appears as the K.E of the electron, i-e

Incident photon energy − Work function = $(K.E)_{max}$ of photoelectron

or

$$\boxed{hf - \phi = \frac{1}{2}mv^2_{max}}$$

This is known as Einstein's photoelectric equation.

When $(K.E)_{max}$ of photoelectron is zero, the frequency is equal to threshold frequency f_o, then above eq. becomes

$$hf_o - \phi = 0$$

or

$$\phi = hf_o$$

∴ Einstein's photoelectric equation can be written as

$$hf - hf_o = (K.E)_{max}$$

or

$$\boxed{hf - hf_o = \frac{1}{2}mv^2_{max} = eV_o}$$

Interms of wavelength

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_o} = \frac{1}{2}mv^2_{max} = eV_o$$

$$\left(\because c = f\lambda, \quad \frac{c}{\lambda} = f\right)$$

where λ_o is the threshold critical or cut off wavelength of photon. It is the longest wavelength at which photoelectrons are emitted.

Note:- It is to be noted that all the emitted electrons dont possess the max. K.E., some electrons come straight out of the metal and some lose energy in atomic collisions before coming out. The above equations holds good only for those electrons which come out with full surplus energy.

<u>Nobel Prize</u> Albert Einstein was awarded Nobel Prize in physics in 1921 for his explanation of photoelectric effect.

(vii) Conclusion

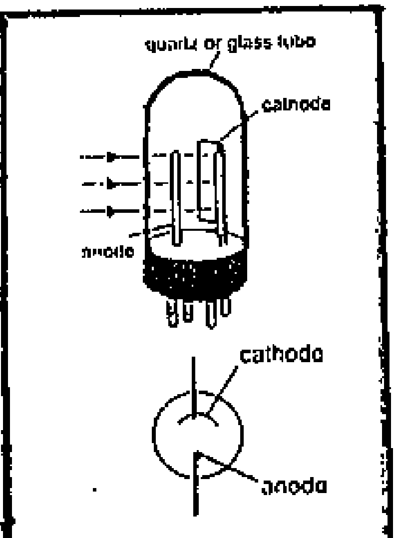
The phenomenon of photoelectric effect cannot be explained if we assume that light consists of waves and energy is uniformly distributed over its wave-front. It can only be explained by assuming light consists of corpuscles of energy known as photon. And each photon is absorbed by a single electron. Thus it shows the corpuscular (particle) nature of light.

(c) PHOTOCELL

(i) **Definition** :- A photocell is a device which converts light energy into electrical energy.

(ii) **Principle** :- Its working principle is based on photoelectric effect.

(iii). **Construction** :- A simple photocell is shown in figure. It consists of an evacuated glass tube containing a thin anode rod and a cathode of some suitable metal surface. The material of the cathode is selected to suit to the frequency range of incident radiation over which the cell is operated.



Simple photo-emissive cell

For example: Sodium or potassium cathode emits electrons for visible light. Cesium coated oxidized silver emits electrons for infrared light and some other metals respond to ultraviolet radiation.

(iv). **Working** :- When light of suitable frequency falls on cathode plate, electrons are emitted and a current flows in the external circuit. This current increases with the increase in intensity of the light. The current stops flowing when incident light beam is interrupted.

(v). Applications :- The cell has wide range of applications. Some of these are given below:

- Security Systems
- Counting Systems
- Automatic door Systems
- Automatic Street lighting
- Exposure meter for photography
- Sound track of movies

(as shown in figures below)

Fig Sound track on a film which varies the intensity of light reaching the photo cell.

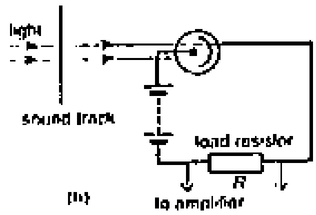


Fig. Photocell detection circuit for sound track of movies.

Example 19.6 :- A sodium surface is illuminated with light of wavelength 300 nm. The work fn. of sodium metal is 2.46 eV.

(a). Find the max. K.E of the ejected electron.

(b). Determine the cut off wavelength for sodium.

Data :- $\lambda = 300\,nm = 300 \times 10^{-9}\,m$

$\phi = 2.46\,eV$

$(K.E)_{max} = ?$

$\lambda_o = ?$

Solution :-

(a) – Energy of incident photon $E = hf = \frac{hc}{\lambda}$

$$E = \frac{6.63\times10^{-34}\ Js \times 3\times10^{8}\ ms^{-1}}{300\times10^{-9}\ m} = 6.63\times10^{-19}\ J$$

$$E = \frac{6.63\times10^{-19}}{1.6\times10^{-19}}\ eV = 4.14\ eV$$

Now $(K.E)_{max} = E - \phi = 4.14\ eV - 2.46\ eV$

$$\boxed{(K.E)_{max} = 1.68\ eV}$$

(b)- $\phi = 2.46\ eV = 2.46\times1.6\times10^{-19}\ J$

$\phi = 3.94\times10^{-19}\ J$

Using $\phi = hf_o = hc/\lambda_o$

$$\lambda_o = hc/\phi = \frac{6.63\times10^{-34}\ Js \times 3\times10^{8}\ ms^{-1}}{3.94\times10^{-19}\ J}$$

$$\lambda_o = 5.05\times10^{-7}\ m$$

$$\boxed{\lambda_o = 505\times10^{-9}\ m = 505\ nm}$$

The cutoff wavelength is in the green region of the visible spectrum.

2. COMPTON EFFECT

(a)- **Introduction** :- Arthur Holly Compton at Washington University in 1923 studied the scattering of X-rays by loosely bound electrons from a graphite target (Fig. a). He measured the wavelength of X-rays scattered and introduced this effect.

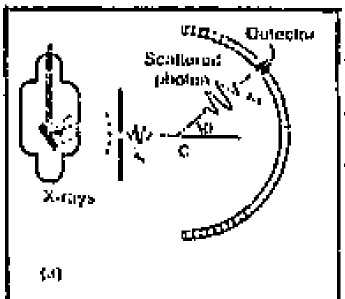


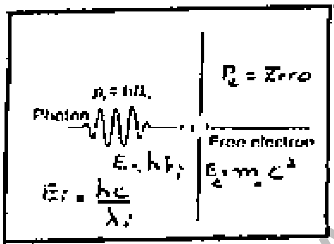
Fig.: (a) Compton's scattering experiment

(b) - **Definition** :- The phenomenon in which the wavelength λ_s of the scattered X-rays is larger than the wavelength λ_i

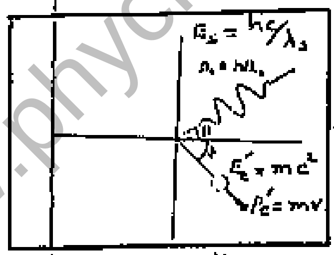
of the incident X-rays is known as Compton effect. $(\lambda_s > \lambda_i)$

(C). Explanation :-

(i). The increase in wavelength of scattered X-rays could not be explained on the basis of classical wave theory. Compton suggested that X-rays consist of photons and in the process of scattering the photons suffer collision with electrons like billiard balls. In this collision, a part of incident photon energy and momentum is transferred to an electron. By applying energy and momentum conservation laws to the process, he derived an expression for the change in wavelength $\Delta\lambda$ known as compton shift.



Fig(b) A photon collides with an electron



Fig(c) Both are scattered

(ii) Expression For Compton Shift $(\Delta\lambda)$

Let x-ray photon of energy $\frac{hc}{\lambda_i}$ and momentum h/λ_i suffers an elastic collision with an electron of graphite with rest mass energy $m_o c^2$ and momentum zero. The incident photon transfers some of its energy to the electron and is scattered with remaining energy hc/λ_s and momentum h/λ_s at an angle 'θ' to its initial direction of motion as shown in fig (c). The electron at rest on getting energy from incident photon is also scattered with energy mc^2 and momentum mv at an angle ϕ to the initial direction of motion of photon as shown in fig (c).

Since the collision is elastic, so both energy and momentum should be conserved. 31

Conservation of Energy

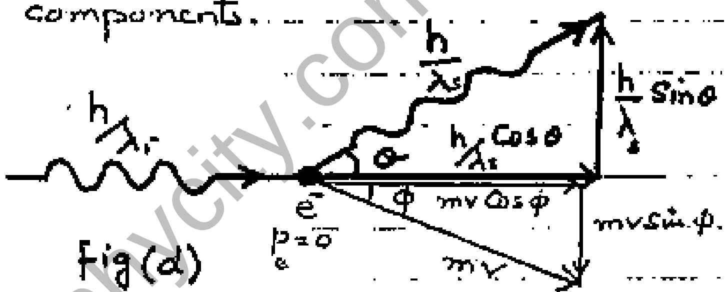
Energy before collision = Energy after collision

$$\frac{hc}{\lambda_i} + m_0c^2 = \frac{hc}{\lambda_s} + mc^2$$

or $$\frac{hc}{\lambda_i} - \frac{hc}{\lambda_s} = mc^2 - m_0c^2 \quad \text{——— (1)}$$

Conservation of Momentum

Since momentum is a vector quantity so we resolve it into its rectangular components as shown in fig(d)



Fig(d)

Along horizontal direction

$$\frac{h}{\lambda_i} + 0 = \frac{h}{\lambda_s}\cos\theta + mv\cos\phi$$

$$\frac{h}{\lambda_i} - \frac{h}{\lambda_s}\cos\theta = mv\cos\phi \quad \text{——— (2)}$$

Along vertical direction

$$0 + 0 = \frac{h}{\lambda_s}\sin\theta - mv\sin\phi$$

$$\frac{h}{\lambda_s}\sin\theta = mv\sin\phi \quad \text{——— (3)}$$

We know that

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad \text{——— (4)}$$

By solving above equations we get

$$\lambda_s - \lambda_i = \frac{h}{m_0c}(1 - \cos\theta)$$

or $$\boxed{\Delta\lambda = \frac{h}{m_0c}(1 - \cos\theta)}$$ where m_0 = rest mass of electron

(iii). Compton Wavelength

Def:- The factor h/m_oc has dimensions of length and is called Compton wavelength.

i.e $\boxed{\text{Compton wavelength} = h/m_oc}$

Numerical Value :- Its numerical value is

$$\frac{h}{m_oc} = \frac{6.63\times10^{-34}\,Js}{9.1\times10^{-31}kg\times3\times10^{8}ms^{-1}}$$

$$= \boxed{2.43\times10^{-12}\,m}$$

→ If the scattered X-ray photons are observed at $\theta = 90°$, the Compton shift $\Delta\lambda$ equals the Compton wavelength.

(iv) - **Conclusion** :- The equation $\Delta\lambda = \frac{h}{m_oc}(1-\cos\theta)$ was found to be in complete agreement with Compton's experimental result, which again is a striking confirmation of particle like interaction of electromagnetic waves with matter.

(v) **Nobel Prize** :- Arthur Holly Compton was awarded Nobel Prize in physics in 1927 for his discovery of the effect named after him.

Example 19.7 :- A 50 keV photon is Compton scattered by a quasi-free electron. If the scattered photon comes off at $45°$, what is its wavelength?

Data :- $E_i = 50\text{ keV} = 50\times10^{3}\times1.6\times10^{-19}\,J$

$\theta = 45°$

$\lambda_s = ?$

Solution :- Using $E = hf = \frac{hc}{\lambda}$ or $\lambda_i = \frac{hc}{E_i}$

$$\lambda_i = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}ms^{-1}}{80\times10^{-16}\,J} = 0.0248\times10^{-9}m$$

$$\lambda_i = 0.0248\text{ nm}$$

Now $\Delta\lambda = \lambda_s - \lambda_i = \frac{h}{mc}(1-\cos\theta)$

$$\lambda_s - \lambda_i = \frac{6.63\times10^{-34}\,Js}{9.1\times10^{-31}\,kg\times3\times10^{8}\,ms^{-1}}(1-\cos 45^\circ)$$

$$= 0.2429\times10^{-11}\,m \times 0.293$$

$$\lambda_s - \lambda_i = 0.0007\,nm$$

or $\lambda_s = \lambda_i + 0.0007\,nm = 0.0248\,nm + 0.0007\,nm$

$$\boxed{\lambda_s = 0.0255\,nm}$$

③. PAIR PRODUCTION

(a) Introduction :-

A photon has zero restmass and all its mass is due to its K.E. It interacts with the matter in different ways

(i)- In case of photoelectric effect, a low energy photon interacting with a metal is usually completely absorbed with the emission of electron.

(ii)- In case of Compton effect, a high energy photon such as that of X-rays is scattered by an atomic electron Transferring a part of its energy to the electron.

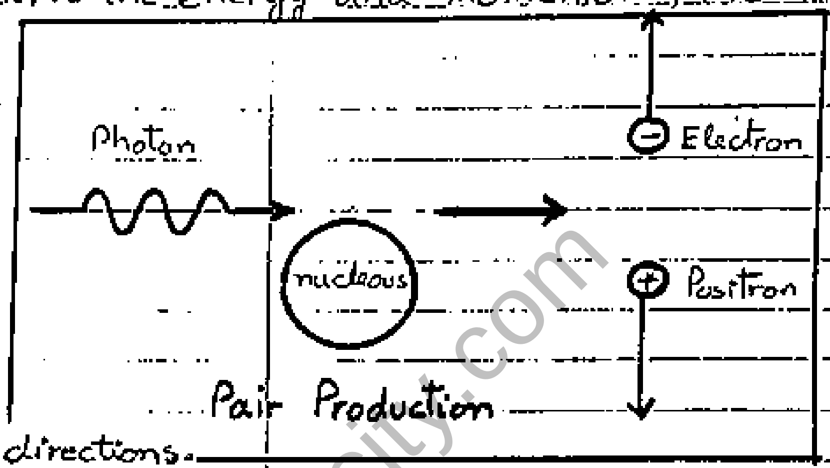
(iii)- A third kind of interaction of very high energy photon such as that of γ-rays with matter is pair production in which photon energy is changed into an electron - positron pair.

(b). Definition :-

When there is an interaction of very high energy photon (such as that of γ- rays) with matter, then this photon disintegrates into an electron - positron pair. This phenomenon is known as Pair Production.

(c) - Explanation :-

(i) - A positron is a particle having mass and charge equal to that of electron but the charge being of opposite nature i-e positive. The creation of two particles with equal and opposite charges is essential for charge conservation in the universe. The positron is also known as antiparticle of electron or anti-electron. In order to conserve the energy and momentum, the pair production takes place in the electric field in the vicinity of a heavy nucleous which takes the recoil and the created particles are moved in opposite directions.



(ii) - Materialization Of Energy:

In this process, radiant energy is converted into matter in accordance with Einstein's equation $E = mc^2$, hence is also known as materialization of energy.

(iii) - Condition For Pair Production:

For an electron or positron, the restmass energy is

$$m_o c^2 = 9.1 \times 10^{-31}\,kg \times (3 \times 10^8\,ms^{-1})^2 = 0.511\,MeV$$

Thus to create the two particles $2m_oc^2 = 2 \times 0.511\,MeV$ or $1.02\,MeV$ energy is required.

For photons of energy greater than $1.02\,MeV$, the probability of pair production occurrence increases as energy increases and the surplus energy is carried off by the two particles in the form of K.E.

(iv) - Equation For Pair Production:

The process can be represented by the equation

$$\text{Energy of photon} = \left[\begin{array}{c}\text{Energy required for}\\ \text{pair production}\end{array}\right] + \left[\begin{array}{c}\text{K.E of the}\\ \text{particles}\end{array}\right]$$

$$\boxed{hf = 2m_oc^2 + (K.E)_{e^-} + (K.E)_{e^+}}$$

§ 19.6 ANNIHILATION OF MATTER

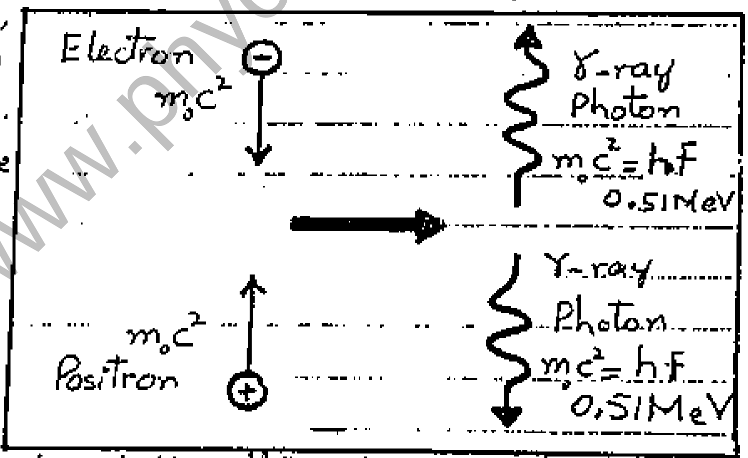
1- Definition :- The reverse process of pair production is known as annihilation of matter.

Or When an antiparticle comes close to a particle, they annihilate and produce two photons in the γ-rays range. This process is called annihilation of matter.

2. Explanation :- When an electron and positron combine together, they annihilate or destroy eachother and the matter (mass) of these two particles appears as electromagnetic energy in the form of two gamma rays photons.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

In this process charge, momentum and energy all are conserved.



(i) **Charge :-** The charge is conserved as the net charge on the pair is zero and γ-ray photons are electrically neutral.

(ii) **Momentum :-** The two photons are produced travelling in opposite directions (as shown in fig) so that momentum is conserved.

(iii) **Energy :-** The rest mass energy of the two electrons ($2m_oc^2 = 1.02$ MeV) converts into two photons. Each photon has energy 0.51 MeV equivalent to rest mass energy of a particle.

NOTE These two phenomenae (pair prodn. and annihilation of matter) tell that energy can be converted into matter and matter into energy, therefore they provide an experimental verification for Einstein's mass - energy relation

$$E = mc^2$$

3- ANTIMATTER

(i) Definition :- A substance which annihilates the matter when it comes across it and gives birth to energy is called antimatter.

(ii) Explanation :- The existance of positron shows the existence of anti-matter. The existance of positron was predicted by Dirac in 1928 and it was discovered in the cosmic radiation in 1932 by Carl Anderson.

It gradually became clear that every particle has a corresponding antiparticle with the same mass and charge (if it is a charged particle) but of opposite sign. Particles and antiparticles also differ in the signs of other quantum numbers.

A particle and its antiparticle cannot exist together at one place. Whenever they meet they annihilate each other. That is, the particle and antiparticle disappear, their combined rest energies appear in other forms.

In 1955 antiproton was discovered by Segre and Chamberlain. Evidence of antineutron followed rapidly in 1956 completing the basic electron, proton and neutron "Antitrio."

Proton and antiproton annihilation has also been observed at Lawrence Berkeley Laboratory.

§ 19.7 WAVE NATURE OF PARTICLES

① Introduction :- It has been observed that light displays a dual nature, it acts as a wave and it also acts as a particle. In 1924, the French physicist, Louis de Broglie thought that if waves can sometimes act like particles, perhaps then particles might act like waves under certain circumstances. So a new type of a wave was introduced named as de Broglie wave.

② Definition of de Broglie waves:-

"The waves associated with the moving particles are called as de Broglie waves or matter waves or particle waves."

③ Explanation :-

In this case particles such as electrons can behave like photons which are emitted and absorbed as discrete bundles of energy i.e electrons can carry their energy by executing wave motion.

(a). de Broglie Relation :-

As momentum p of photon is given by equation

$$p = \frac{h}{\lambda}$$

de Broglie suggested that momentum of a material particle of mass m moving with velocity v should be given by the same expression.

$$p = mv = \frac{h}{\lambda}$$

or

$$\boxed{\lambda = \frac{h}{mv}}$$

Do You Know?

Light is, in short, the most refined form of matter (Louis de Broglie 1892-1987).

Where λ is the wavelength associated with particle waves. The above equation is called de Broglie relation.

(b)- Applications of de Broglie equation

By using this relation and after finding the wavelength of the wave associated with moving object, it can be said that which one particle can has wave like nature and which one has not.

For Examples :-

(i)- A rifle bullet of mass 20g and flying with speed 330 ms^{-1} will have a wavelength λ given by

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,Js}{2\times10^{-2}\,kg\times330\,ms^{-1}} = 1\times10^{-34}\,m$$

This wavelength is so small that it is not measurable or detectable by any of its effects.

Therefore it is concluded that

"An object of large mass and ordinary speed has such a small wavelength that its wave effects such as interference and diffraction are negligible. so such objects have not wave like nature."

(ii)- For an electron moving with a speed of $1\times10^{6}\,ms^{-1}$

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,Js}{9.1\times10^{-31}\,kg\times1\times10^{6}\,ms^{-1}} = 7\times10^{-10}\,m$$

This wavelength is in the X-rays range.

Hence an electron can be considered to be a particle and it can also be considered to be a wave.

Note:- The diffraction effects for electrons are measurable where as diffraction or interference effects for bullets are not.

(C)- Experimental Verifications.

(i)- Davisson And Germer Experiment

The first experimental proof of wave nature of electrons was provided by Clinton J. Davisson and Laster H. Germer. They showed that electrons are diffracted from metal crystals in exactly the same manner as X-rays or any other wave.

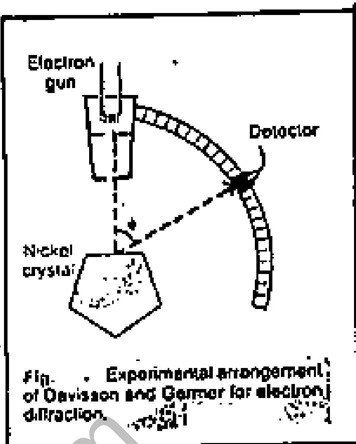


Fig. Experimental arrangement of Davisson and Germer for electron diffraction.

The apparatus used by them is shown in figure in which electrons from heated filament are accelerated by an adjustable applied voltage V.

The energy gained by the electron beam

$$K.E = Ve.$$

This beam is made incident on a nickel crystal. The beam diffracted from crystal surface enters a detector and is recorded as current I. The gain in K.E of the electron as it is accelerated by a potential V in the electron gun is given by

$$\frac{1}{2}mv^2 = Ve$$

or

$$mv^2 = 2Ve$$

$$m^2v^2 = 2mVe$$

$$mv = \sqrt{2mVe}$$

Experimentally Observed Wavelength

Using above eq. in de Broglie relation

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mVe}}$$

In one of the experiments, the accelerating voltage V was 54 volts, hence:

$$\lambda = \frac{6.63\times10^{-34}\,Js}{\sqrt{2\times9.1\times10^{-31}\,kg\times54\,JC^{-1}\times1.6\times10^{-19}\,C}}$$

$$\boxed{\lambda = 1.66\times10^{-10}\,m}$$

This was the experimentally observed wavelength by Davisson and Germer.

Theoretically Predicted Wavelength

This beam of electrons diffracted from crystal surface was obtained for a glancing angle of 65°.

According to Bragg's equation (From X-ray diffraction)

$$2d\sin\theta = m\lambda$$

For 1^{st} order diffraction $m = 1$

For nickle $d = 0.91\times10^{-10}m$

Thus

$$2\times0.91\times10^{-10}m \sin 65^\circ = 1\times\lambda$$

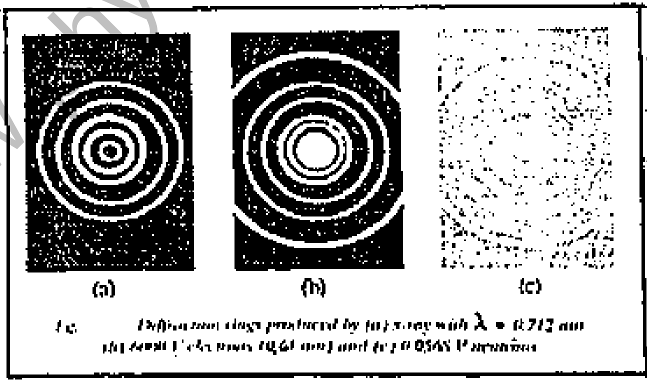
$$\boxed{\lambda = 1.65\times10^{-10}m}$$

Thus experimentally observed wavelength is in excellent agreement with theoretically predicted wavelength.

(ii). **G. P. Thomson Experiment :-**

Another experimental proof was given by George Paget Thomson (son of J.J. Thomson) of England, in which he observed the diffraction by passing x-rays and a beam of electrons of approximately the same wavelength through thin aluminium foil as shown in figure below.

(iii). Diffraction pattern have also been observed with protons, neutrons, hydrogen atoms and helium atoms thereby giving substantial evidence for the wave nature of particles.



(d) - **Nobel Prizes**

→ For the work on the dual nature of particles, Prince Louis Victor de Broglie received the Nobel prize in 1929 in physics.

→ Clinton Joseph Davisson and George Paget Thomson shared the Nobel Prize in 1937 for their experimental confirmation of the wave nature of particles.

(e) Wave Particle Duality

(i) Introduction :-

Dual nature of light : Interference, diffraction and polarization of light confirm its wave nature while photoelectric effect and compton effect prove the particle nature of light.

Dual nature of electron beam : The experiments of Davisson, Germer and G. P. Thomson reveal wave like nature of electrons and in the experiment of J. J. Thomson to find e/m we had to assume particle like nature of the electron.

In the same way we are forced to assume both wavelike and particle like properties for all matter (electrons, protons, neutrons, molecules etc) and also light (ultra violet radiations, X-rays, γ-rays etc)

(ii). Definition :-

Matter and radiation have a dual 'wave-particle' nature this new concept is known as wave particle duality.

(iii). Explanation :-

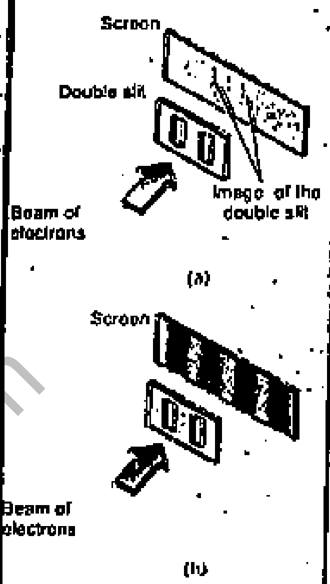
Neil Bohr's Principle of Complementarity

Neil Bohr pointed out in stating his principle of complementarity that both wave and particle aspects are required for the complete description of both radiation and matter. These both aspects cannot be revealed simultaneously in a single experiment, which aspect is revealed is determined by the nature of the experiment being done.

For Example: If we perform an experiment with diffraction grating we are treating light as a wave. However if we make a beam of light to fall on a metal surface, we regard it as a stream of particles to explain our observations.

There is no simple experiment that you can carry out with the beam that will require you to interpret it as a wave and as a particle at the same time.

Light behaves as a stream of photons when it interacts with matter and behaves as a wave in traveling from a source to the place where it is detected.



(a) If electrons behaved as discrete particles with no wave properties, they would pass through one or the other of the two slits and strike the screen, causing it to glow and produce exact images of the slits. (b) In reality the screen reveals a pattern of bright and dark fringes, similar to light is used and interference occurs between the light waves coming from each slit.

(iv). Conclusion :-

All micro-particles (electrons, protons, neutrons, photons, atoms etc) propagate as they were waves and exchange energies as if they were particles - that is the wave particle duality.

(f) Uses Of Wave Nature Of Particles

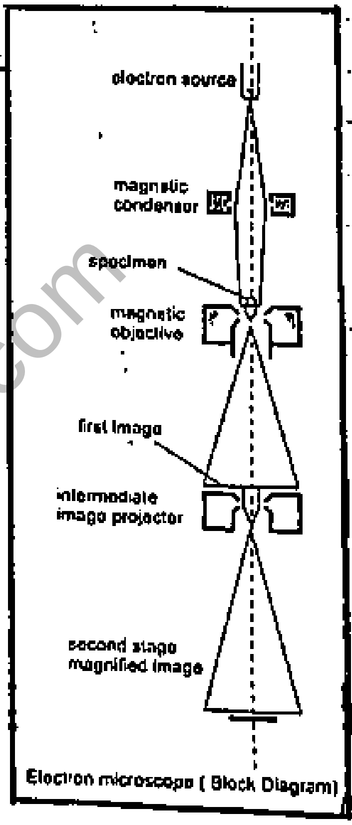
The fact that energetic particles have extremely short de Broglie wavelengths is brought into practical use in many a latest ultra-modern devices of great importance such as Electron Microscope.

ELECTRON MICROSCOPE

(i). Definition An instrument that uses a beam of electrons to investigate a sample in order to achieve a higher magnification and resolution than is possible with an optical microscope.

(ii). **Principle** :- Working principle of an electron microscope is based on the wave-nature of moving electrons.

(iii). **Schematic Diagram** :-
A schematic diagram of the electron microscope is shown in fig.



Electron microscope (Block Diagram)

(iv). **Construction** :-
In an electron microscope, electric and magnetic fields rather than optical lenses are used to focus electron by means of electromagnetic forces that are exerted on moving charges. The resulting deflections of the electrons beams are similar to the refraction effects produced by glass lenses used to focus light in optical microscope. The electrons are accelerated to high energies by applying voltage from 30 kV to several megavolts. Such high voltages give extremely short wavelength and also give the electron sufficient energy to penetrate specimen of reasonable thickness.

(V). **Working** :- The magnetic conducting lens (magnetic condenser) concentrates the beam from an electron gun on to the specimen. Electrons are scattered out of the beam from the thicker parts (nuclei) of the specimen. The transmitted beam therefore has spatial differences in density that correspond to the features of the specimen. The objective and intermediate lenses produce a real intermediate image and projection lens forms the final image which can be viewed on a fluorescent screen or photographed on a special film known as **electron micrography**.

(vi)- Magnifying Power :- Since the wavelength associated with an electron is thousand of time shorter than the wavelength of visible light, so its magnifying power is far greater than that of optical microscope. It is the wave nature of electrons which makes the electron microscope to show minor details not visible with optical microscope.

(vii)- Resolution :- A resolution of 0.5 to 1 nm is possible with a 50 kV microscope as compared to best optical resolution of 0.2 μm.

(viii)- Scanning Electron Microscope :-
A three dimensional image of remarkable quality can be achieved by modern versions called scanning electron microscopes.

x ———————— x

Example 19.8: A particle of mass 5.0 mg moves with speed of 8.0 ms^{-1}. Calculate its de-Broglie wavelength.

Data:-
$m = 5.0\,mg = 5.0 \times 10^{-6}\,kg$
$v = 8.0\,ms^{-1}$
$h = 6.63 \times 10^{-34}\,Js$
$\lambda = ?$

Solution:- Using

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}\,Js}{5.0 \times 10^{-6}\,kg \times 8.0\,ms^{-1}}$$

$$\boxed{\lambda \doteq 1.66 \times 10^{-27}\,m}$$

Example 19.9:- An electron is accelerated through a Potential Difference of 50V. Calculate its de Broglie wavelength.

Data:-

$m = 9.1 \times 10^{-31}\,kg$

$V_o = 50V$

$e = 1.6 \times 10^{-19}C$

$\lambda = ?$

Solution:-

Using law of conservation of energy

loss in electrical P.E = gain in K.E

$$V_o e = \frac{1}{2} m v^2$$

$$2 V_o e = m v^2$$

multiplying on both sides by m

$$2 m V_o e = m^2 v^2$$

or

$$\sqrt{2 m V_o e} = m v = p$$

Now using de Broglie relation

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\lambda = \frac{h}{\sqrt{2 m V_o e}}$$

$$\lambda = \frac{6.63 \times 10^{-34}\,Js}{\sqrt{2 \times 9.1 \times 10^{-31}\,kg \times 50V \times 1.6 \times 10^{-19}C}}$$

$$\boxed{\lambda = 1.74 \times 10^{-10}\,m}$$

S:- 19.8 UNCERTAINTY PRINCIPLE

1. Introduction :-

Position and momentum of a particle cannot both be measured simultaneously with perfect accuracy even with an ideal instrument. There is always a fundamental uncertainty associated with any measurement. This uncertainty is not related to the measuring instrument but it is the result of wave - particle duality of matter and radiation.

This principle was first proposed by Werner Heisenberg in 1927 on the basis of dual nature of light and hence is known as Heisenberg Uncertainty Principle.

This fundamental uncertainty is completely negligible for measurements of position and momentum of macroscopic objects but in case of microscopic objects i.e subatomic particles, the uncertainty is not negligible.

Example :- A stream of light scattering from a flying tennis ball does not change its path. But one photon striking an electron changes its motion.

2. Statement :-

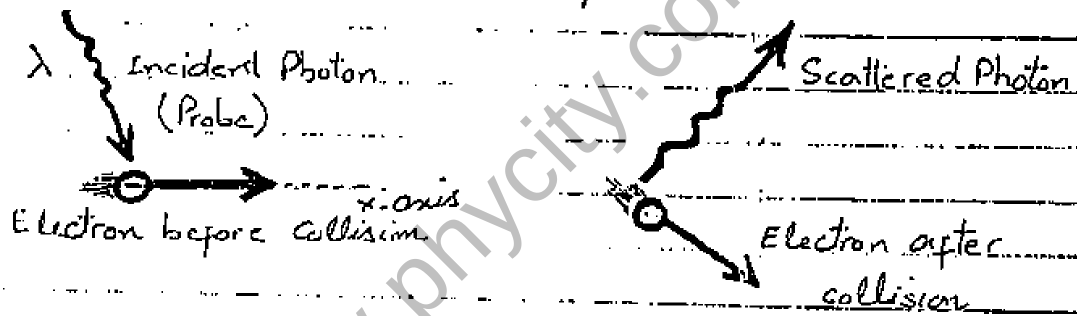
This principle states that the product of the uncertainty in the position (Δx) of a particle at some instant and the uncertainty in the x-component of its momentum (Δp) at the same instant approximately equals to Planck's constant (h).

Mathematically :-

$$\Delta x \Delta p \simeq h$$

3. Explanation :-

(a) Suppose we want to measure the position and momentum of an electron at a given moment. We may look at the electron by using light of wave length λ. When one photon of light called 'Probe' strikes an electron or micro particle moving along x-axis, the photon will be scattered and the original position and momentum of the electron will be changed. Hence, in order to observe the position of an electron with less uncertainty and also for minimizing diffraction effect, we must use light of short wavelength. But it will alter the motion drastically making momentum measurements less precise.



If light of wavelength λ is used to locate a micro-particle (electron) moving along x-axis, the uncertainty in its position measurement is

$$\Delta x \approx \lambda \quad \text{——— (1)}$$

At most, the photon of light can transfer all its momentum $\left(\frac{h}{\lambda}\right)$ to the microparticle whose own momentum will then be uncertain by an amount

$$\Delta p \approx h/\lambda \quad \text{——— (2)}$$

Equation (1) shows that in order to reduce the uncertainty in position we must use light of shorter wavelength where as equation (2) shows that in order to reduce the uncertainty in momentum, we must use light of longer wavelength, so at an instant only one quantity can be predicted accurately.

Multiplying eq ① and ②

$$\Delta x . \Delta p \approx \lambda \left(\frac{h}{\lambda} \right)$$

$$\boxed{\Delta x . \Delta p \approx \lambda} \quad \text{③}$$

This equation gives the mathematical form of Uncertainty Principle.

For Your Information

You can never accurately describe all aspects of a subatomic particle at once.

Do You Know?

In the subatomic world few things can be predicted with 100% precision.

(b) Another Form Of Uncertainty Principle

When there will be uncertainty in the momentum of the particle, then its energy will also be uncertain.

$$\left(p = E/c \Rightarrow \Delta p = \Delta E/c \right)$$

The energy E of a photon of frequency f is given by

$$E = hf$$

If ΔE is the uncertainty in measurement of energy of a particle in time interval Δt then

$$\Delta E = h \Delta f$$

but $\Delta f = \frac{1}{\Delta t}$ so

$$\Delta E = h \left(\frac{1}{\Delta t} \right)$$

Alternative Method

If $t_o + \frac{\Delta t}{2}$ is the timings during which the particle is expected to have energy between E and $E + \Delta E/2$ then $\Delta E \Delta t \approx h$

$$\boxed{\Delta E \, \Delta t \approx h} \quad \text{④}$$

This equation gives another form of uncertainty principle

Another Statement:

The product of the uncertainty in a measured quantity of energy (ΔE) and the uncertainty in interval of time (Δt) during which it is measured is approximately equal to Planck's const (h).

Equation ④ shows that more accurately we determined the energy of a particle, the more uncertain we will be of the time during which it has that energy.

(c). Heisenberg's Accuracy in Calculations

According to Heisenberg's more careful calculations, he found that at the very best

$$\Delta x . \Delta p \geq \hbar \quad \text{——— (5)}$$

and

$$\Delta E . \Delta t \geq \hbar \quad \text{——— (6)}$$

Where

$$\hbar = \frac{h}{2\pi} = \frac{6.63 \times 10^{-34}\ Js}{2 \times 3.14}$$

$$\hbar = 1.05 \times 10^{-34}\ Js$$

The factor $\hbar = h/2\pi$ is used to express spin of elementary particles.

(d). Nobel Prize :-

Werner Karl Heisenberg received Nobel Prize for physics in 1932 for the development of quantum mechanics.

(e). Uses of Heisenberg Uncertainty Principle

(i). This principle tells us that we cannot know everything about a particle. There is uncertainty in momentum at a given position and uncertainty in energy at a given instant.

(ii). It can be used to justify wave-particle duality.

(iii). It can explain why the electrons cannot exist inside the nucleus of an atom.

e.g

Example # 19.11

An electron is to be confined to a box of the size of the nucleus (1.0×10^{-14} m). What would the speed of the electron be if it were so confined?

Data :- $\Delta x = 1.0 \times 10^{-14}$ m , $m = 9.1 \times 10^{-31}$ kg

$\hbar = 1.05 \times 10^{-34}$ Js , $\Delta V = ?$

Calculation :- Maximum uncertainty in the location of electron equals to the size of the box itself that is $\Delta x = 1.0 \times 10^{-14}\, m$. The minimum uncertainty in the velocity of electron is found from Heisenberg uncertainty principle.

$$\Delta x \, \Delta p \approx \hbar$$

or

$$\Delta p \approx \hbar / \Delta x$$

or

$$m \Delta v \approx \hbar / \Delta x$$

$$\Delta v \approx \frac{\hbar}{m \Delta x}$$

$$\Delta v = \frac{1.05 \times 10^{-34}\, Js}{9.1 \times 10^{-31}\, kg \times 1.0 \times 10^{-14}\, m}$$

$$\Delta v = 1.15 \times 10^{10}\, m s^{-1}$$

Conclusion :- For confinement in the box, the speed of electron should be greater than the speed of light. Because, this is not possible, we must conclude that an electron can never be found inside the nucleus.

Example 19.10 :- The life time of an electron in an excited state is about $10^{-8}\, s$. What is its uncertainty in energy during this time?

Data :- $\Delta t = 10^{-8}\, s$, $\hbar = 1.05 \times 10^{-34}\, Js$, $\Delta E = ?$

Calculations :- Using uncertainty principle

$$\Delta E \, \Delta t \approx \hbar$$

$$\Delta E = \hbar / \Delta t$$

$$= \frac{1.05 \times 10^{-34}\, Js}{10^{-8}\, s}$$

$$\Delta E = 1.05 \times 10^{-26}\, J$$