

PROBLEMS CH # 17

P#17.1 A 1.25 cm diameter cylinder is subjected to a load of 2500 kg. Calculate the stress on the bar in mega pascals.

SOLUTION

$$d = 1.25 \text{ cm} = 1.25 \times 10^{-2} \text{ m}$$

$$r = \frac{d}{2} = \frac{1.25}{2} \times 10^{-2} \text{ m} = 6.25 \times 10^{-3} \text{ m}$$

$$r = 6.25 \times 10^{-3} \text{ m}$$

$$m = 2500 \text{ Kg}$$

$$\text{Stress} = \sigma = ?$$

$$\therefore \sigma = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{2500 \times 9.8}{3.14 \times (6.25 \times 10^{-3})^2}$$

$$\sigma = 199.7 \times 10^6 \text{ Pa}$$

$$= 199.7 \text{ Mpa}$$

$$\approx 200 \text{ Mpa}$$

★ ★ ★ ★

P# 17.2 A 1.0 m long copper wire is subjected to stretching force and its length increases by 20 cm. Calculate the tensile strain and the percent elongation which the wire undergoes.

SOLUTION

$$l = 1 \text{ m}$$

$$\Delta l = 20 \text{ cm} = 0.2 \text{ m}$$

$$\% \text{ elongation} = ?$$

$$\text{Tensile strain} = ?$$

$$\% \text{ elongation} = \frac{\Delta l}{l} \times 100$$

$$= \frac{0.2}{1} \times 100 = 20\%$$

$$\text{Tensile strain} = \frac{\Delta l}{l}$$

$$= \frac{0.2}{1}$$

★ ★ ★ ★

P # 17.3 A wire 2.5 m long and cross-section area 10^{-5} m^2 is stretched 1.5 mm by a force of 100 N in the elastic region. Calculate

- (i) strain
- (ii) Young's Modulus
- (iii) The energy stored in the wire.

SOLUTION

$$l = 2.5 \text{ m}$$

$$A = 10^{-5} \text{ m}^2$$

$$\Delta l = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$F = 100 \text{ N}$$

- (i) Strain = $\epsilon = ?$
- (ii) Young's Modulus = $Y = ?$
- (iii) Energy stored = ?

$$(i) \quad \epsilon = \frac{\Delta l}{l} = \frac{1.5 \times 10^{-3}}{2.5} = 6 \times 10^{-4}$$

$$(ii) \quad Y = \frac{\sigma}{\epsilon} = \frac{F/A}{\epsilon} = \frac{F}{A\epsilon}$$

$$= \frac{100}{10^{-5} \times 6 \times 10^{-4}} = 1.66 \times 10^{10} \text{ N/m}^2 \text{ or pascal.}$$

$$(iii) \quad E = \frac{1}{2} \frac{YA \Delta l^2}{l}$$

$$= \frac{1}{2} \left[\frac{1.66 \times 10^{10} \times 10^{-5} \times (1.5 \times 10^{-3})^2}{2.5} \right]$$

$$= 0.075 \text{ J} = 7.5 \times 10^{-2} \text{ J}$$

★ ★ ★ ★

P # 17.4 What stress would cause a wire to increase in length by 0.01%. If the Young's modulus of the wire is 12×10^{10} pascals, what force would produce this stress if the diameter of the wire is 0.56 mm.

SOLUTION

$$\% \text{ elongation} = 0.01\%$$

Date: P # 32

$$Y = 12 \times 10^{10} \text{ Pa}$$

$$d = 0.56 \text{ mm} = 0.56 \times 10^{-3} \text{ m} = 5.6 \times 10^{-4} \text{ m}$$

$$r = \frac{d}{2} = 2.8 \times 10^{-4} \text{ m}$$

$$F = ?$$

$$\% \text{ elongation} = 0.01 \%$$

$$\frac{\Delta l}{l} \times 100 = 0.01$$

$$\frac{\Delta l}{l} = \frac{0.01}{100} = 10^{-4}$$

$$\epsilon = 10^{-4}$$

$$\therefore \text{Young's Modulus} = Y = \frac{\sigma}{\epsilon}$$

$$\Rightarrow \sigma = Y \epsilon = 12 \times 10^{10} \times 10^{-4} = 12 \times 10^6 \text{ Pa}$$

$$\text{As } \sigma = F/A$$

$$\Rightarrow F = \sigma A = \sigma \pi r^2 = 12 \times 10^6 \times 3.14 \times (2.8 \times 10^{-4})^2 = 2.95 \text{ N}$$

★ ★ ★ ★

P # 17.5 The length of steel wire is 1.0m and its cross-sectional area is $0.03 \times 10^{-4} \text{ m}^2$. Calculate the work done in stretching the wire when a force of 100N is applied within the elastic region. Young's Modulus of steel is $3.0 \times 10^{11} \text{ Nm}^{-2}$

SOLUTION

$$l = 1 \text{ m}$$

$$A = 0.03 \times 10^{-4} \text{ m}^2$$

$$F = 100 \text{ N}$$

$$Y = 3 \times 10^{11} \text{ Nm}^{-2}$$

$$\epsilon = ?$$

$$\text{Work} = ?$$

$$\therefore Y = \frac{\sigma}{\epsilon} = \frac{F/A}{\Delta l/l}$$

$$Y = \frac{F l}{A \Delta l}$$

$$\Rightarrow \Delta l = \frac{F l}{A Y}$$

$$= \frac{100 \times 1}{0.03 \times 10^{-4} \times 3 \times 10^{11}}$$

$$\Delta l = 1.11 \times 10^{-4} \text{ m}$$

$$\text{Work} = \frac{1}{2} \frac{Y A (\Delta l)^2}{l}$$

$$= \frac{1}{2} \times 3 \times 10^{11} \times 0.03 \times 10^{-4} \times (1.11 \times 10^{-4})^2$$

$$= 5.5 \times 10^{-3} \text{ J}$$

OR

$$W = \langle F \rangle \Delta l = \frac{1}{2} F \times \Delta l$$

$$= \frac{1}{2} \times 100 \times 1.11 \times 10^{-4}$$

$$= 5.55 \times 10^{-3} \text{ J}$$

* * * *

P# 17.6 A cylindrical Copper wire and a cylindrical steel wire each of length 1.5m and diameter 2.0mm are joined at one end to form a composite wire 3.0m long. The wire is loaded until its length becomes 3.003m. Calculate the strain in Copper and steel wires and force applied to the wire. (Young's Modulus of Copper is 1.2×10^{11} pascal and for steel is 2.0×10^{11} Pa.)

SOLUTION

$$l_c = l_s = 1.5 \text{ m}$$

$$d_c = d_s = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$r = 1 \times 10^{-3} \text{ m}$$

$$\text{Initial length} = l = l_c + l_s = 1.5 + 1.5 = 3.0 \text{ m}$$

$$\text{Final length} = l' = l + \Delta l = 3.003 \text{ m}$$

$$\Rightarrow \Delta l = l' - l = 3.003 - 3.0$$

$$= .003 \text{ m}$$

CH # 17

Date: P # 34

$$Y_c = 1.2 \times 10^{11} \text{ Pa}$$

$$Y_s = 2 \times 10^{11} \text{ Pa}$$

$$E_s = ? \quad E_c = ?$$

$$F = ?$$

$$\therefore Y = \frac{\sigma}{\epsilon}$$

$$\Rightarrow Y_c = \frac{\sigma_c}{\epsilon_c} \quad \& \quad Y_s = \frac{\sigma_s}{\epsilon_s}$$

$$\sigma_c = Y_c \epsilon_c, \quad \sigma_s = Y_s \epsilon_s$$

As both the wires experience the same stress

$$\therefore \sigma_c = \sigma_s$$

$$Y_c \epsilon_c = Y_s \epsilon_s$$

$$Y_c \left(\frac{\Delta l}{l} \right)_c = Y_s \left(\frac{\Delta l}{l} \right)_s$$

$$Y_c \frac{\Delta l_c}{l_c} = Y_s \frac{\Delta l_s}{l_s} \quad (\because l_c = l_s)$$

$$Y_c \Delta l_c = Y_s \Delta l_s$$

Let $\Delta l_c = x$

then $\Delta l_s = 0.003 - x$ ($\because \Delta l_c + \Delta l_s = 0.003 \text{ m}$)

Hence $Y_c x = Y_s (0.003 - x)$

$$x = \frac{Y_s (0.003 - x)}{Y_c}$$

$$= \frac{2 \times 10^{11} (0.003 - x)}{1.2 \times 10^{11}}$$

$$x = 1.66 (0.003 - x)$$

$$x = 4.98 \times 10^{-3} - 1.66x$$

$$x + 1.66x = 4.98 \times 10^{-3}$$

$$2.66x = 4.98 \times 10^{-3}$$

$$\Delta l_c = x = 1.872 \times 10^{-3} \text{ m}$$

$$E_c = \frac{\Delta l_c}{l_c} = \frac{1.872 \times 10^{-3}}{1.5}$$

$$= 1.248 \times 10^{-3} = 1.25 \times 10^{-3}$$

$$E_s = \frac{\Delta l_s}{l_s} = \frac{.003 - x}{l_s} = \frac{.003 - 1.872 \times 10^{-3}}{1.5}$$

$$E_s = 7.52 \times 10^{-4}$$

$$E_s = .75 \times 10^{-3}$$

$$\therefore \text{stress} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow \text{Force} = \text{Stress} \times \text{Area}$$

$$= \sigma_c \times A_c$$

$$= Y_c \times E_s \times \pi r_c^2$$

$$= 1.2 \times 10^{11} \times 1.25 \times 10^{-3} \times 3.14 \times (1 \times 10^{-3})^2$$

$$= 471.2 \text{ N}$$

SHORT QUESTIONS

For short Questions from 17.1 to 17.10

See Theory

S.Q 17.11 What is meant by hysteresis loss.

How it is used in the construction of a transformer?

Ans # Energy needed to magnetize and demagnetize the material in each loop is called by hysteresis loss

In transformer we use a core made up of soft copper strips in order to minimize the hysteresis loss and to improve the efficiency of the transformer.