CHAPTER — 1 : (1)

MEASUREMENTS

INTRODUCTION :-

The earlier observations of man about the world around him and facts about the natural phenomena and material things resulted in the birth of single discipline of science called "natural philosophy".

Due to huge increase in the scientific knowledge uptill the begining of 19th Century, it was found necessory to classify the study of nature into two main branches as given below :

(i) **Biological Sciences** :-

"The sciences which deals with the study of living things are called Biological sciences."

Examples : Biology, Zoology, Botany, Physiology etc.

(ii) **Physical Sciences** :-

"The sciences which deals with non-living things are called Physical sciences."

Examples : Physics, Chemistry, Astronomy, Geology etc.

The science and scientific method emphasizes the need of accurate measurements of various measureable features. This chapter emphasizes the need of thorough understanding and practice of measuring techniques and recording skills.

INTRODUCTION TO PHYSICS :-

"Physics is an important and basic part of physical sciences." It is an experimental science, required and applicable to nearly all disciplines of science.

At present time, there are following three main frontiers of fundamental science:

(i) The world of extremely large body — the universe itself — its birth and expansion by "**THE CREATOR**", from a very special "Cosmic egg" with temp. of millions of degrees (Big-Bang theory), which probably started 20 billion years ago.

(ii) The world of extremely small particles such as, electrons, protons, neutrons, mesons and other sub-atomic particles.

(iii) The world of complex matter. It is also the world of "middle sized" things, from molecules at one extreme, bodies on the earth and the earth at the other extreme. This is fundamental Physics, which is the heart of Science.

WHAT IS PHYSICS?

"It is the branch of science which deals with the study of matter, energy and the relationship between them".

The study of physics involves investigating such things as the laws of motion, structure of space and time, the nature and type of forces that hold different materials together, the interaction between different particles, the interaction of electromagnetic radiation with matter and so on.

By the end of 19th century many physicists started believing that every thing about physics has been discovered. However, about the begining of the twentieth century many new experimental facts revealed that the laws formulated by the previous investigators need modifications. Further researches gave birth to many new disciplines in physics.

In short, "Physics upto 1890 A.D. is called classical physics" and "Physics after 1890 A.D uptill now and onwards is called modern physics".

A few branches of "Classical Physics" are:

(i) **Mechanics**:- "This branch of physics deals with the motion of particles under the action of different forces".

(ii) **Heat and thermodynamics** :- "This branch of physics deals with the nature of heat, its measurement, measurement of temp. and conversion of heat energy into mechanical energy".

(iii) **Electromagnetism** :- "This branch deals with interaction of electricity and magnetism, their development and use in practical life".

(IV) **Optics** :- "This branch deals with the study of nature of light, its properties and about optical instruments of different types".

(V) **Sound** :- "This branch deals with production nature and different properties of sound."

(VI) **Hydrodynamics** :- This branch deals with the study of the motion of Fluids (liquids and gases).

A few sub-branches of modern physics are :-

(i) **Special relativity** :- In this branch we study about non-accelerated frames of reference. This was stated by Einstein in 1905.

(ii) **General relativity** :- In this branch we study about the accelerated frames of reference. This theory came in 1915 (by Einstein).

(iii) **Quantum mechanics** :- It is the mathematical theory based on Plank's quantum hypothesis.

(IV) **Atomic Physics** :- This branch deals with behaviour, structure and nature of atoms.

(V) **Molecular Physics** :- In this branch we study about the structure of matter on the basis of molecules.

(VI) **Nuclear Physics** :- In this branch we study about the atomic nuclei, nuclear reactions and radioactivity.

(VII) **Solid State Physics** :- This branch is concerned with the structure and properties of solids.

(VIII) **Particle Physics** :- This branch is concerned with 'elementary particles' of matter.

(IX) **Super Conductivity** :- This branch deals with the study of super conductors and their development.

(X) **Superfluidity** :- This branch deals with the behaviour and nature of super-fluids.

(XI) **Plasma Physics** :- This branch is concerned with highly ionized gases at very high temperatures. Plasma is called fourth state of matter.

(XII) **Magnetohydrodynamics** :- This branch deals with the behaviour of a conducting fluid under the influence of a magnetic flux.

(XIII) **Space Physics** :- This branch deals with the different aspects of space.

Some other related branches are:

Astrophysics, Biophysics, Chemical physics, Engineering physics, Geophysics, medical physics, physical oceanography, physics of music.

Physics is most fundamental of all sciences and provides other branches of science, basic principles and fundamental laws. This overlaping of physics gave birth to new branches such as physical chemistry, biophysics, astrophysics, health physics etc. Physics also plays an important role in the development of technology and engineering.

We are living in the age of information technology. The information media and fast means of Communications have made the world as a "Global village".

PHYSICAL QUANTITIES :-

Defi. :- "Those quantities which can be measured accurately are called physical quantities".

The foundation of physics rests upon physical quantities.

For example: mass, length, time, velocity, force, density, temp., electric current etc.

TYPES :- Physical quantities are of two types:
(i) Base quantities (ii) Derived quantities.

(i) Base Quantities :- "These are the minimum number of those physical quantities in terms of which other physical quantities can be defined."
For example: length, mass, time

(ii) Derived Quantities :- "The quantities whose definitions are based on other physical quantities are called derived quantities."
For example: velocity, acceleration, force etc.

"The measurement of a base quantity" involves following two steps:

(i) The choice of a standard

(ii) Establishment of a procedure for comparing the quantity to be measured with the standard.

An ideal standard has two characteristics:

(a) It is accessible,

(b) It is invariable.

INTERNATIONAL SYSTEM OF UNITS :

In 1960, an international committee agreed on a set of definitions and standards to describe the physical quantities. The system that was established is called the System International (SI).

Due to simplicity and convenience this system of units is being used by the world's scientific community and by most nations.

The **SI-units** are build up from three kinds of units :

(A) Base units (B) Supplimentary units

(C) Derived units.

(A) Base units :- "The units defined arbitrarily for the measurement of seven base quantities in comparison with them are called base units."

The names of base units along with respective physical quantities and symbols are given below:

Sr. No.	Physical Quantity	SI-Unit	Symbol
1.	Length	metre	m
2.	Mass	Kilogram	kg
3.	Time	Second	s
4.	Electric current	ampere	A
5.	Temperature	Kelvin	K
6.	Light intensity	candela	cd
7.	Amount of Substance	mole	mol

The standard definitions of base units are given below:

(1) Metre :-

The unit of length is metre. Before 1960 it was defined as:

Defi.—1 : "The distance between two lines marked on the bar of an alloy of platinum (90%) and iridium (10%) kept under controlled conditions at the International Bureau of Weights and Measures in France."

The 11th General Conference on Weights and Measures (1960) redefined the standard metre as follows:

Defi.—2 :- "one metre is a length equal to 1,650,763.73 wavelengths in vacuum of the orange red radiation emitted by the Krypton-86 atom."

In 1983, the metre was redefined as:

Defi.—3 :- "It is the distance travelled by light in vacuum during a time of $\frac{1}{299,792,458}$ second."

A few 'sub-multiples' and 'multiples' of metre are:

(i) 1 micrometer $= 10^{-6}$ m	(i) 1 decameter $= 10^{1}$ m
(ii) 1 millimeter $= 10^{-3}$ m	(ii) 1 hectometer $= 10^{2}$ m
(iii) 1 Centimeter $= 10^{-2}$ m	(iii) 1 Kilometer $= 10^{3}$ m
(IV) 1 decimeter $= 10^{-1}$ m.	(IV) 1 megameter $= 10^{6}$ m.

* (i) 10 mm = 1 cm (ii) 10 cm = 1 decimeter
(iii) 1000 mm = 1 m (IV) 1000 m = 1 Km.

(2) Kilogram :-

The unit of mass is Kilogram. It is defined as: "The mass of a platinum (90%) and iridium (10%) alloy cylinder, 3.9 cm in diameter and 3.9 cm in height, kept at the International Bureau of Weights and Measures in France."

This mass standard was established in 1901.

A few conversion relations of 'kg' are:

(i) 1000 milligram = 1 gram

(ii) 1000 gram = 1 Kilogram

(iii) 100 Kilogram = 1 Quintal

(IV) 1000 Kilogram = 1 metric tonne.

(3) Second :-

The unit of time is termed as 'second'.

Defi.1:- "one second is equal to $\frac{1}{86400}$ part of an average day of the year 1900 A.D."

In 1967, an International Committee redefined second as:

Defi. 2 :- "one second is equal to the duration in which the outer most electron of the Cesium-133 atom makes 9192,631,770 vibrations."

A few sub-multiples and multiples of 'second' are as given below:

(i) 1 nanosecond = 10^{-9} s	(i) 60 s = 1 min.
(ii) 1 micro " = 10^{-6} s	(ii) 60 min. = 1 hr.
(iii) 1 milli " = 10^{-3} s	(iii) 24 hrs. = 1 day.

* (a) 365 days = 1 year, (b) 10 years = 1 decade.

(c) 100 years = 1 Century (d) 1000 years = 1 Millennium.

(4) **Kelvin** :-

The unit of temperature is Kelvin.

Defi. :- "It is the fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the tripple point of water."

The tripple point of a substance means the temp. at which solid, liquid and vapour phases are in equilibrium. The tripple point of water is taken as 273.16 K. This standard was adopted in 1967.

(5) **Ampere** :-

The unit of electric current is ampere.

Defi. :- "one ampere is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed a metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length."

This unit was established in 1971.

(6) **Candela** :-

The unit of light intensity (or luminous intensity) is candela, which is defined as:

Defi. :- "one candela is the luminous intensity in the perpendicular direction of a surface of $\frac{1}{600,000}$ square meter of a black body radiator at the solidification temperature of platinum under standard atmospheric pressure".

This definition was adopted in 1967.

(7) Mole :-

The unit of "amount of substance" is mole.

Defi. :- "one mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of Carbon-12." This standard was adopted in 1971.

When the unit 'mole' is used, the elementary entities must be specified. These may be atoms, molecules, ions, electrons, other particles or specified groups of such particles. One mole of any substance contains 6.0225×10^{23} entities.

(B) Supplementary units :-

Those SI-units which are not included either in base units or in derived units are called supplementary units. These are two in number as given below:

Sr. No.	Physical Quantity	SI-Unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	Steradian	sr

Definitions :-

1. Radian :- "The radian is the plane angle between two radii of a circle which cut off on the circumference an arc, equal in length

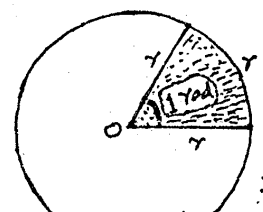

Fig-1.

to the radius of the circle. (fig.-1).

Total angle of the circumference = 2π rad.

2. Steradian :-

"The steradian is the solid angle (three-dimensional angle) subtended at the centre of a sphere by an area of its surface equal to the square of radius of the sphere. (Fig.-2).

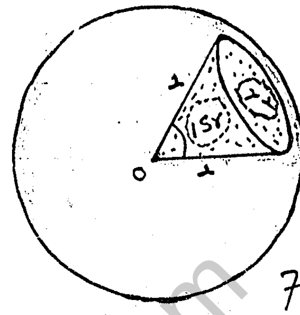

Fig.-2.

(C) Derived Units :-

"SI-units for measuring all other physical quantities are derived from the base and supplementary units. These are called derived units. Some of the derived units are given as under:

Sr.No.	Physical Quantity	Unit	Symbol	In terms of base units
1.	Force	newton	N	$kg\ m\ s^{-2}$
2.	Work	joule	J	$N\ m = kg\ m^2 s^{-2}$
3.	Power	watt	W	$J\ s^{-1} = kg\ m^2 s^{-3}$
4.	Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1} s^{-2}$
5.	Electric charge	coulomb	C	A s

etc. etc.

Scientific Notation :-

"When the numbers are expressed in standard form, it is called scientific notation, which employs powers of ten."

Rule :- while writing the numbers in scientific notation, there should be only one non-zero digit to the left of decimal.

Examples :-

Sr. No.	Simple Notation	Scientific Notation
1.	134.7	1.347×10^{2}
2.	0.0023	2.3×10^{-3}
3.	1225	1.225×10^{3}

etc.

Conventions for indicating units :-

Following points should be kept in mind while using units :

(i) Full name of the unit does not begin with a capital even if named after a scientist, e.g. newton.

(ii) The symbol of unit named after a scientist has initial capital letter such as 'N' for newton.

(iii) The prefix should be written before the unit without any space, such as: 1×10^{-3} m = 1 mm

(iv) A Combination of units is written each with one space apart. e.g. newton metre = N m.

(v) Compound prefixes are not allowed. e.g. 1 μμF may be written as 1 pF.

(vi) A number such as 5.0×10^{4} cm may be expressed in scientific notation as 5.0×10^{2} m.

(vii) when a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone. Thus: $1\,Km^2 = 1(Km)^2 = 1\times10^6\,m^2$.

(viii) Measurement in practical work should be recorded immediately in the most convenient unit, e.g. reading of screw gauge in 'mm', mass in 'grams'. But before calculation for the result, all measurements must be converted into SI-units.

* Imp. Prefixes :-

(i)	atto = 10^{-18}	(a)		(IX)	deca = 10^{1}	(da)
(ii)	femto = 10^{-15}	(f)		(X)	Hecto = 10^{2}	(h)
(iii)	pico = 10^{-12}	(p)		(XI)	Kilo = 10^{3}	(K)
(IV)	nano = 10^{-9}	(n)		(XII)	mega = 10^{6}	(M)
(V)	micro = 10^{-6}	(μ)		(XIII)	giga = 10^{9}	(G)
(VI)	milli = 10^{-3}	(m)		(XIV)	tera = 10^{12}	(T)
(VII)	Centi = 10^{-2}	(c)		(XV)	peta = 10^{15}	(P)
(VIII)	deci = 10^{-1}	(d)		(XVI)	exa = 10^{18}	(E)

ERRORS AND UNCERTAINITIES :-

Every measurement has some error to some extent. It is very difficult to elliminate all possible errors or uncertanities in a measurement. The error in a measurement may occur due to:

(i) Negligence or inexperience of a person.

(ii) The faulty apparatus.

(iii) Inappropriate method or technique.

The uncertainity may also occur due to inadequacy of an instrument, natural variations or natural imperfactions of a person's senses. However, there are two major types of errors:

(i) Random error (ii) Systematic error

(i) Random error :-

This error is said to occur when repeated measurements of the quantity, give different values under the same conditions. e.g. different values of time period, different values of 'g' etc.

This error can be reduced by repeating the measurements several times and taking an average.

(ii) Systemic error :-

This error refers to an effect that influences all measurements of a particular quantity equally. It produces a consistent difference in readings.

It may occur due to zero error, poor calibration of instruments etc.

This error can be reduced by comparing a faulty instrument with a more accurate one.

Thus for systematic error, a correction factor can be applied. e.g. to remove zero error we apply zero correction.

SIGNIFICANT FIGURES :-

Defi. :- "In any measurement, the accurately known digits and first doubtful digit are called significant figures."

In other words, a significant figure is the one known to be reasonably reliable.

Explanation with examples :-

We know that every measuring instrument is calibrated to a certain smallest division which limits its degree of accuracy.

Example-1 :- Suppose that we want to measure the length of a straight line with the help of a metre rod calibrated in millimetres. Let the end point of the line lies between 10.3 and 10.4 cm marks as shown in fig.-3.

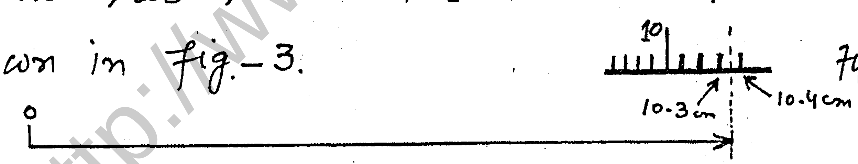

Fig.-3

By convention, if the end of the line does not touch or cross the midpoint of the smallest division, the reading is confined to the previous division (10.3 cm).

In case the end of the line seems to be touching or have crossed the midpoint, the reading is extended to the next division (10.4 cm).

Example-2 :- By applying the above rule (example-1) the position of the edge of a line recorded as 12.7 cm with the help of a metre rod calibrated in millimetres may lie between 12.65 cm and 12.75 cm. Thus in this example the max. uncertainty is ±0.05 cm. It is in fact equivalent to half of the <u>least count</u> of metre rod (<u>1mm = 0.1cm</u>) i.e. $\pm\frac{1}{2}(0.1cm) = \pm 0.05$ cm. i.e. half above and half below the recorded reading.

<u>**Significance**</u> :- The uncertainty in the value of measured quantity can be indicated conveniently by using significant figures. The recorded value of the length of straight line i.e. 12.7 cm contains three digits (1, 2, 7) out of which two digits (1, 2) are accurately known digits while the third digit i.e. 7 is a doubtful one. Thus by definition, the no. of significant figures in the given value i.e. 12.7 cm are 'three'.

If the above measurement (i.e. 12.7 cm) is taken by a better measuring instrument which is exact upto a hundreth of 'cm', it would have been recorded as : 12.70 cm. In this case, the no. of significant figures are "four".

<u>**Conclusion**</u> :- Thus we can say that as we improve the quality of our measuring instrument and techniques, we extend the measured result to more and more significant figures and correspondingly improve the experimental accuracy of the result.

Rules of significant Figures :-

(i) All the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are significant. However, zeros may or may not be significant. ∴ In case of zeros, the rules are:

(a) A zero between two significant figures is itself significant. e.g. 102, 10.5 etc.

(b) Zeros to the left of significant figures are not significant. e.g. none of the zeros 0.00467, 02.59 is significant.

(c) Zeros to the right of a significant figures may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. e.g. all the zeros in 3.570, 7.4000 are significant. However, in integers such as 8,000 kg, the no. of significant zeros is determined by the accuracy of the measuring instrument. If the measuring scale has a least count of '1 kg' then there are 'four' significant figures written in scientific notation as 8.000×10^3 kg. If least count of the scale is 10 kg, then the no. of significant figures will be 'three' when written in scientific notation as 8.00×10^3 kg and so on.

(d) when a measurement is recorded in scientific notation, the figures other than the powers of ten are significant figures. e.g. a measurement recorded as 8.70×10^{4} kg has 'three' significant figures.

(ii) "<u>In multiplying or dividing numbers,</u> keep a number of significant figures in the result equal to the factor containing the least no. of significant figures."

For example : $\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.4576898 2 \times 10^{3}$

As the factor "3.64×10^{4}" in the above calculation has the least no. of significant figures i.e. three, so the answer should have only 'three' significant figures. So the answer is : 1.46×10^{3}.

<u>The other figures in the above calculation are insignificant and are deleted/dropped according to the following rules:</u>

(a) If the dropping digit is less than '5', the retaining digit will remain unchanged.

(b) If the dropping digit is greater than five, the retaining digit will be increased by one.

(c) If the digit to be dropped is 5, the retaining digit is increased by one if it is odd and retained as such if it is even.

For example, the following numbers are

rounded off to three significant figures as follows:

43.75	is rounded off as	43.8
56.8546	" " " "	56.9
73.650	" " " "	73.6
64.350	" " " "	64.4

(iii) "<u>In adding or subtracting numbers</u>, the no. of decimal places retained in the answer should be equal to the smallest no. of decimal places in any of the quantities being added or subtracted."

Examples :- (a)

$$\begin{array}{r} 72.1 \text{ m} \\ 3.42 \text{ "} \\ 0.003 \text{ "} \\ \hline 75.523 \text{ "} \end{array}$$

(b)

$$\begin{array}{r} 2.7543 \text{ m} \\ 4.10 \text{ "} \\ 1.273 \text{ "} \\ \hline 8.1273 \text{ "} \end{array}$$

Correct Answer: 75.5 m   8.13 m

The above 'rule' is applied in calculating correct answers in examples (a) and (b). ∴

<u>PRECISION AND ACCURACY</u> :-

<u>Precision</u> :- "The precision of a measurement is determined by the instrument (or device) being used."

A precise measurement is that one which has less absolute uncertainty (or error).

<u>Accuracy</u> :- "The accuracy of a measurement depends upon the fractional or percentage uncertainty in that measurement."

An accurate measurement is that one which has less fractional (or percentage) uncertainty.

<u>Absolute Uncertanity</u> :- "Absolute uncertainty (or error) is equal to the least count of the measuring instrument."

Fractional error :- "It is the ratio of absolute error to the quantity measured." i.e.

$$\therefore \text{Fractional error} = \frac{\text{Absolute error}}{\text{quantity}}$$

Example-1 :-

Let the length of an object is recorded as 25.5cm by using a metre rod, having smallest division in millimetres. The uncertainty in the measurement is ±0.05. on doubling it we get $(\pm 0.05 \times 2) = \pm 0.1$ cm which is called absolute uncertainty and is equal the least count of metre rod. So in the above measurement:

$\therefore$ Absolute uncertainty $= \pm 0.1$ cm

Fractional " $= \frac{0.1\,\cancel{cm}}{25.5\,\cancel{cm}} = 0.0039 = 0.004$

Percentage " $= \frac{0.1\,\cancel{cm}}{25.5\,\cancel{cm}} \times \frac{100}{100} = \frac{0.4}{100} = 0.4\%$

Example-2 :-

Let us take another measurement using vernier callipers with least count 0.01cm is recorded as 0.45cm.

$\therefore$ It has : Absolute error $= \pm 0.01$ cm

Fractional " $= \frac{0.01\,\cancel{cm}}{0.45\,\cancel{cm}} = 0.022 = 0.02$

Percentage " $= \frac{0.01\,\cancel{cm}}{0.45\,\cancel{cm}} \times \frac{100}{100} = \frac{2}{100} = 2\%$

Conclusion :- It is concluded on the basis of results of example-'1' and -'2' that : "The reading 25.5cm taken by metre rod is more accurate (or less precise) due to having less %age error (0.4% < 2%) whereas the reading 0.45cm taken by vernier callipers is more precise (or less accurate) due to having less absolute error (0.01cm < 0.1 cm)."

ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT :-

The maximum possible uncertainty (or error) in the final result can be found by evaluating the uncertainties in all the factors involved in that calculation. In this regard the rules are as under:

1. For addition and subtraction :-

Rule :- "In addition and subtraction absolute uncertainties are added".

Example :- The distance 'x' is determined by the difference between two seperate position measurements 'x_1' and 'x_2' where: $x_1 = 10.5 \pm 0.1$ cm

$x = ?$ $\qquad$ $x_2 = 26.8 \pm 0.1$ cm.

Sol. :- $\because x = x_2 - x_1$ $\qquad \therefore$

$$\begin{aligned} x_2 &= 26.8 \pm 0.1 \text{ cm} \\ x_1 &= 10.5 \pm 0.1 \text{ cm} \\ \hline \end{aligned}$$

$$\therefore x = 16.3 \pm 0.2 \text{ cm}.$$

* It is to be noted here that the quantities are subtracted but by rule uncertainties are added.

2. For multiplication and division :-

Rule :- "In multiplication and division percentage uncertainties are added".

Example :- The max. possible error in the value of resistance 'R' of a conductor determined from the Potential diff. 'V' and current 'I' by using formula "$R = \frac{V}{I}$" is found as follows:

Sol.:- As given: $V = 5.2 \pm 0.1$ volts

$I = 0.84 \pm 0.05$ ampers

%age error for 'V' $= \frac{0.1\,V}{5.2\,V} \times \frac{100}{100} = 1.92\% = 2\%$

%age " " $I = \frac{0.05\,A}{0.84\,A} \times \frac{100}{100} = 5.95\% = 6\%$

By rule: Total error in the result 'R' $= 2\% + 6\% = 8\%$

$\therefore\ R = \frac{5.2\,V}{0.84\,A} = 6.19\,\Omega$ with an error of 8%

i.e. $R = 6.2 \pm 0.5\,\Omega$

The above result is rounded off to two significant figures, because both 'V' and 'I' have two significant figures.

3. For Power factor :-

Rule :- "Multiply the percentage uncertainty by that power."

Example :- In the calculation of the volume of a sphere using formula: $V = \frac{4}{3}\pi r^3$, r = radius.

%age uncertainty in vol. 'V' = 3 × %age uncertainty in 'r'

If radius, 'r' = 2.25 cm measured by vernier callipers with least count 0.01 cm, then: $r = 2.25 \pm 0.01$ cm

Sol.:- As given: $r = 2.25 \pm 0.01$ cm

$\therefore$ Absolute error (= L.C.) = 0.01 cm

%age error in 'r' $= \frac{0.01\,cm}{2.25\,cm} \times \frac{100}{100} = 0.44\% = 0.4\%$

$\therefore$ Total " " " Vol. 'V' $= 3 \times 0.4 = 1.2\%$

$\therefore\ V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (2.25\,cm)^3$

$= 47.689\,cm^3$ with 1.2% error

$\therefore\ V = 47.7 \pm 0.6\,cm^3$.

4. For uncertainty in the average value of many measurements :-

Rules :- "(i) Find the average value of measured values.

(ii) Find deviation of each measured value from the avg. value.

(iii) The mean deviation is the uncertainty in the average value".

Example :- The six readings of the screw gauge to measure the diameter (d) of a wire in 'mm' are:

1.20, 1.22, 1.23, 1.19, 1.22, 1.21.

Sol. :- using given data: Average 'd' $= \frac{1.20+1.22+1.23+1.19+1.22+1.21}{6}$

$\therefore$ Average 'd' = 1.21 mm

The deviations of readings, which are the differences (regardless of sign) respectively are: 0.01, 0.01, 0.02, 0.02, 0.01, 0

$\therefore$ Mean of deviations (or error) $= \frac{0.01+0.01+0.02+0.02+0.01+0}{6}$

$= 0.01$ mm

$\therefore$ The result is written as:

$$d = 1.21 \pm 0.01 \text{ mm}.$$

5. For the uncertainty in a timing expt.

Rule :- "The uncertainty in the time period of a vibrating body is found by relation:

$$\text{Uncertainty (or error)} = \frac{\text{Least count of S. watch}}{\text{No. of vibrations}}"$$

Example :- Least count of stop watch $= \frac{1s}{10} = 0.1$ s

Time for 30 vib. of simple pendulum = 54.6 s

" " 1 " " " (time period) 'T' $= \frac{54.6}{30} = 1.82$ s

$\therefore$ T = 1.82 s with uncertainty $\frac{0.1s}{30} = 0.003$ s, where

0.1 s is the least count of stop watch.

Thus : $T = 1.82 \pm 0.003$ s.

In order to reduce timing uncertainty one should take large no. of vibrations.

Example 1.1 :- The length, breadth and thickness of a sheat 3.233 m, 2.105 m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.

Sol. :- As given : length 'l' = 3.233 m, Breadth 'b' = 2.105 m,

Thickness 'h' = $1.05\ cm = 1.05 \times 10^{-2}\ m$.

$\therefore$ volume $V = l \times b \times h$

$\therefore V = 3.233 \times 2.105 \times 1.05 \times 10^{-2} = 7.14573825 \times 10^{-2}\ m^3$

As min. no. of significant figures in given factor 1.05×10^{-2} m are three, so : $V = 7.15 \times 10^{-2}\ m^3$.

Example 1.2 :- The mass of a metal box measured by a lever balance is 2.2 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision.

Sol. :- Let: $m = 2.2$ kg, $m_1 = 10.01\ g = 0.01001$ kg

$m_2 = 10.02\ g = 0.01002$ kg.

Total mass : $M = m + m_1 + m_2 = 2.2 + 0.01001 + 0.01002$

$\therefore M = 2.22003$ kg. Since least precise value is 2.2 kg with one decimal place. Hence total mass correct upto appropriate precision is : $M = 2.2$ kg.

Example 1.3 :- The diameter and length of a metal cylinder measured with the help of vernier callipers of least count 0.01 cm are 1.22 cm and 5.35 cm. Calculate the volume 'V' of the cylinder and uncertainty in it.

Sol. :- As given : Diameter 'd' = 1.22 cm, length 'l' = 5.35 cm

L.c. (for V. Callipers) = 0.01 cm, V = ?

Absolute error (= L.c.) = 0.01 cm (for both: l, d)

$$\%\text{age error in length} = \frac{0.01\,cm}{5.35\,cm} \times \frac{100}{100} = 0.2\%$$

$$\text{" " " diameter} = \frac{0.01\,cm}{1.22\,cm} \times \frac{100}{100} = 0.8\%$$

As volume: $V = \pi r^2 l = \pi \left(\frac{d}{2}\right)^2 l$

or $V = \frac{\pi d^2 l}{4}$

Total % age error in 'V' = 2 × (% age error in diameter) + 1 (% age " " length)

$= 2(0.8\%) + 1(0.2\%) = 1.8\%$

Thus : $V = \frac{\pi d^2 l}{4} = \frac{3.14 \times (1.22)^2 \times 5.35}{4} = 6.2509079\ cm^3$ with 1.8 % error

$\therefore V = 6.25 \pm 0.1\ cm^3$

DIMENSIONS OF PHYSICAL QUANTITIES :-

Defi. :- "Each base quantity is considered a dimension denoted by a specific symbol written within square brackets."

Explanation :- A dimension stands for the qualitative nature of a physical quantity. For example, different quantities such as length, breadth, radius.

thickness, diameter, light year etc. all measured in 'metre' are denoted by the dimension of length [L]. Similarly mass is denoted by dimension [M] and time by dimension [T]. other quantities have dimensions which are combinations of these dimensions. For example:

(i) Dimensions of speed $= \frac{\text{Dimension of length}}{\text{Dimension of time}}$

or $[v] = \frac{[L]}{[T]} = [L][T^{-1}]$

$\therefore [v] = [LT^{-1}]$

(ii) Dimensions of acc.: $[a] = [L][T^{-2}] = [LT^{-2}]$

$\therefore [a] = [LT^{-2}]$.

(iii) Dim. of force: $[F] = [m][a] = [M][LT^{-2}]$

$\therefore [F] = [MLT^{-2}]$, and so on.

Dimensional Analysis :- It is a method by which we can check the correctness of a given formula (or an equation) and can also derive it.

* In dimensional analysis we can use dimensions as algebric quantities.

(i) **Checking the homogeneity of Physical Equation** :- In order to check the correctness of an equation, we are to show that the dimensions of the quantities on both sides of the equation are same. This is called the Principle of homogeneity of dimensions.

Example 1.4 :- Check the correctness of the relation : $v = \sqrt{\frac{F\times l}{m}}$, where '$v$' is the speed of transverse waves on a stretched string of tension F, length 'l' and mass 'm'.

Sol. :- As given : $v = \sqrt{\frac{F\times l}{m}}$

Dimensions on L.H.S of eq. $= [v] = [LT^{-1}]$

" " R.H.S " " $= \left\{\frac{[F]\times[l]}{[m]}\right\}^{1/2}$

$$= \left\{[M^1L^1T^{-2}]\times[L^1]\times[M^{-1}]\right\}^{\frac{1}{2}} = \left\{[M^{1-1}][L^{1+1}][T^{-2}]\right\}^{\frac{1}{2}}$$

$$= [L^2T^{-2}]^{1/2} = [LT^{-1}] \qquad \because [M^0]=1.$$

$\therefore$ Dim. on LHS. = Dim. on RHS. Therefore the given eq. is dimensionally correct.

(ii) Deriving a possible formula :-

To derive a relation for a physical quantity depend on the correct guessing of various factors on which that physical quantity depends.

Example 1.5 :- Derive a relation for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period 'T' may depend are:

(i) Length of pendulum (l)

(ii) Mass of the bob (m)

(iii) Angle 'θ' which the thread makes with vertical direction

(IV) Acceleration due to gravity (g).

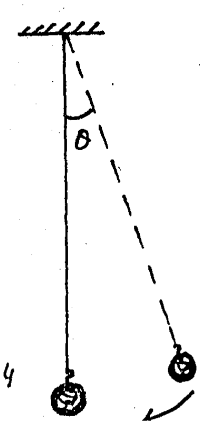

Fig.-4

Sol. :- The relation for the time period 'T' will be of the form:

$$T \propto m^a \times l^b \times \theta^c \times g^d$$

$$\therefore \text{ or } \quad T = \text{Constt.}\left(m^a \times l^b \times \theta^c \times g^d\right) \longrightarrow (1)$$

where we are to find the values of powers a, b, c and d. Writing the dimensions on both sides of above eq., we have:

$$[T] = \text{Constt.}[M]^a \times [L]^b \times [LL^{-1}]^c \times [LT^{-2}]^d$$

$$\text{or} \quad [T] = \text{constt.}[M]^a [L]^{b+\cancel{c}-\cancel{c}+d} [T]^{-2d}$$

$$\text{or} \quad [M]^0[L]^0[T]^1 = \text{constt}[M]^a [L]^{b+d} [T]^{-2d}$$

Equating the powers on both sides, we get:

$$a = 0, \quad b+d = 0, \quad 1 = -2d, \quad d = -\tfrac{1}{2}$$

$$\text{or} \quad a = 0, \quad b = -d = -(-\tfrac{1}{2}) = \tfrac{1}{2}, \quad \therefore b = \tfrac{1}{2}$$

$$\theta = [LL^{-1}]^c = [L^0]^c = [L]^0 = 1 \qquad \left(\because \theta = \frac{s}{r},\ \theta = LL^{-1}\right)$$

putting values of powers in eq. (1) above:

$$T = \text{Constt.}\left(m^0 \times l^{\frac{1}{2}} \times 1 \times g^{-\frac{1}{2}}\right) = \text{constt}\left(\frac{l^{\frac{1}{2}}}{g^{1/2}}\right)$$

$$\therefore T = \text{Constt}\sqrt{\frac{l}{g}}$$

The numerical value of constt. can't be determined by dimensional analysis, however, it can be found by experiments.

Example 1.6 :- Find the dimensions and hence the units of coeff. of viscosity 'η' in the relation of Stoke's law for the drag force 'F' for a spherical object of radius 'r' moving with velocity 'v' given as:

$$F = 6\pi\eta r v.$$

Sol. :- As '6π' in the given formula is a dimensionless number, so it is not accounted in dimensional analysis. Then: $[F] = [\eta r v]$

or $$[\eta] = \frac{[F]}{[r][v]}$$

using the dimensions of F, r, v in RHS.

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} = \frac{[M]^1[L]^1[T]^{-2}}{[L]^2[T]^{-1}}$$

or $$[\eta] = [M]^1[L]^{1-2}[T]^{-2+1}$$

$\therefore$ $$[\eta] = [ML^{-1}T^{-1}]$$

Thus, the SI-unit of Coeff. of viscosity (η) is : $Kg\,m^{-1}s^{-1}$.